PAT

2013

Answers

Part A: Mathematics for Physics [50 Marks]

1. What is the sum of the series $\frac{2}{3} - \frac{2}{9} + \frac{2}{27} - ....?$ [4]

$$S_\infty = \frac{a}{1-r} \quad \text{with} \quad a = \frac{2}{3}, \quad r = -\frac{1}{3}$$

$$= \frac{1}{2}$$

2. If $y = \sqrt{x\nu}$ and $(y - \sqrt{x})^2 = u$,
find an expression for x in terms of u and ν. [3]

$$x = \frac{u}{(\sqrt{v} - 1)^2}$$

3. 50 people are in a room. 8 people in the room have red hair, 3 people have black hair and 20 people are male. You may assume that hair colour and gender are independent.

(a) If a person is selected at random from the room, what is the probability that they will be a female with red hair?

(b) If a person is selected at random from the room, what is the probability that they will be a male who does not have red or black hair?

[3]

a) $\frac{30}{50} \times \frac{8}{50} = \underline{\frac{12}{125}}$

b) $\frac{20}{50} \times \frac{39}{50} = \underline{\frac{39}{125}}$

4. Consider the function $f(x) = x^3 - x^2 - 4x + 4$

(a) Show that $x = 1$ is a root of $f(x) = 0$ and hence factorise $f(x)$ to find the remaining roots.

(b) Having found the roots of $f(x) = 0$, find the area bounded by the curve $f(x)$ and the x-axis between the two smallest roots.

[5]

a) $x=1 \Rightarrow \quad 1^3 - 1^2 - 4\times 1 + 4$

$= 0$ hence root

so $f(x) = (x-1)(x^2-4)$

$x = \pm 2$

b) $$\int_{-2}^{1} x^3 - x^2 - 4x + 4 \, dx$$

$$= \left[\frac{x^4}{4} - \frac{x^3}{3} - 2x^2 + 4x \right]_{-2}^{1}$$

$= 11\frac{1}{4}$

5. If $x = \log_{10} 100 + \log_5 \sqrt{25} - \log_3 y^2$ and
$\frac{x}{2} = \log_2 8 - 9 \log_{10} \sqrt{10} + 2 \log_3 y$

find x and y. [4]

$x = 2 + 1 - 2 \log_3 y$

$= 3 - 2 \log_3 y$

$\frac{3}{2} - \log_3 y = 3 - \frac{9}{2} + 2 \log_3 y$

$\log_3 y = 1$

$\underline{\underline{y = 3}}$ $\quad$ $\underline{\underline{x = 1}}$

6. Find the equation of the straight line that passes through the centres of the two circles: $x^2 + 4x + y^2 - 2y = -1$ and $x^2 - 4x + y^2 - 6y = 3$ [5]

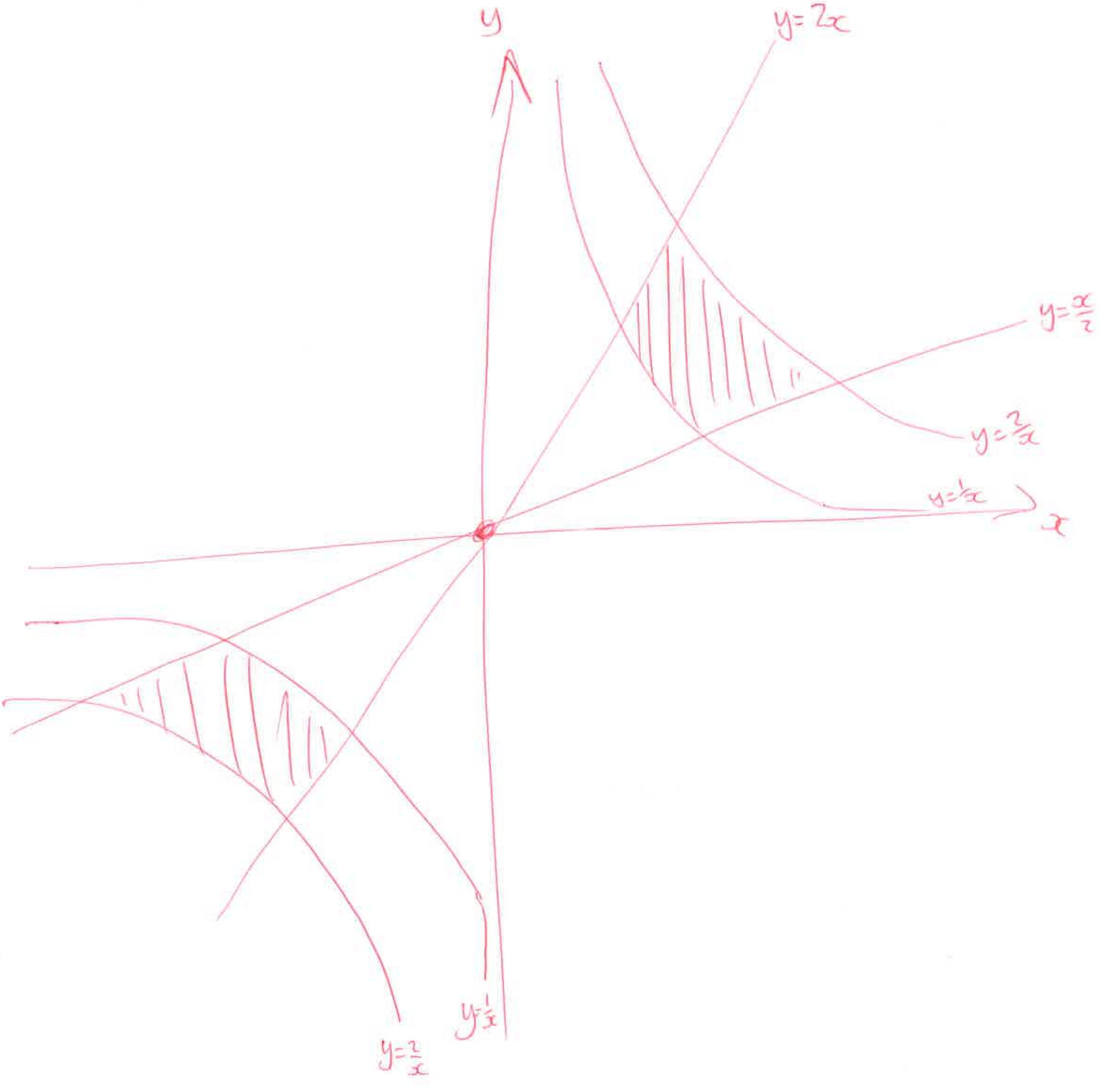
Centres are $(-2, 1)$ and $(2, 3)$

Hence $y = \frac{x}{2} + 2$

7. How many terms in the binomial expansion would be needed to determine $(3.12)^5$ to one decimal place? [4]

4

8. Sketch the regions in the xy plane defined by the inequalities: $1 < xy < 2$ and $\frac{1}{2} < \frac{y}{x} < 2$ [7]

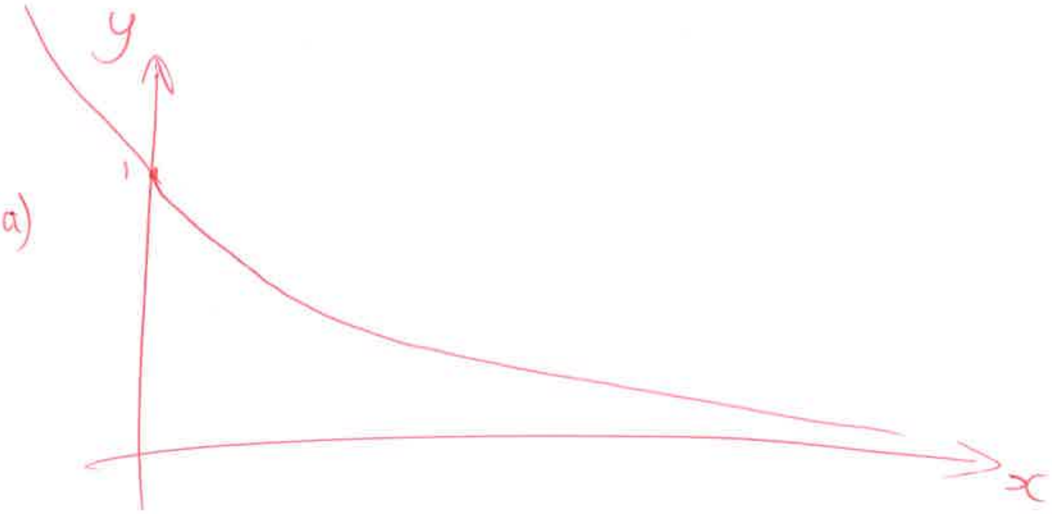


9. (a) Sketch $y = \exp(-x)$

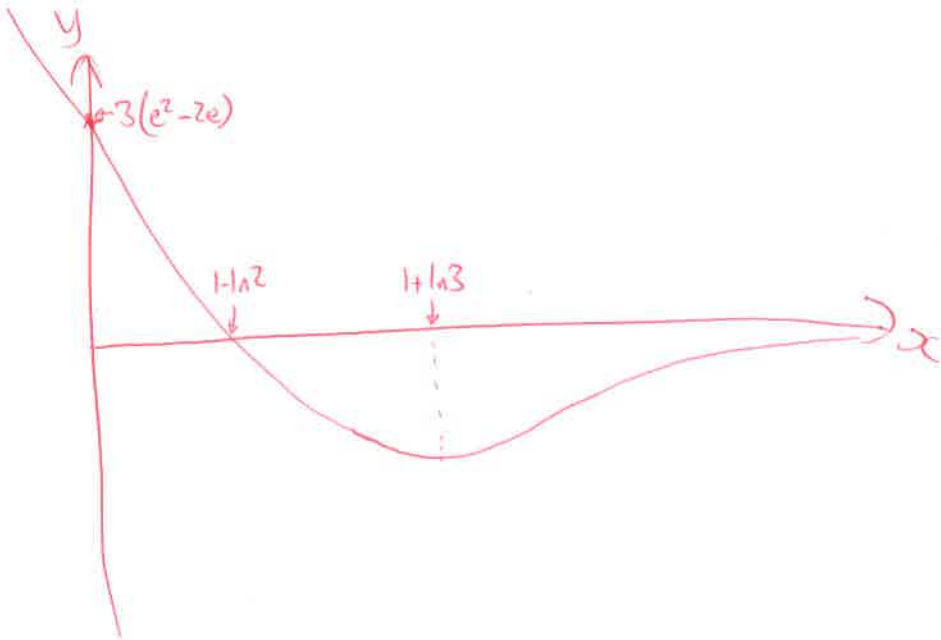
(b) Sketch $y = 3\{\exp[-2(x-1)] - 2\exp[-(x-1)]\}$ for $x > 0$

[$\exp(x)$ is defined as $\exp(x) \equiv e^x$] [8]

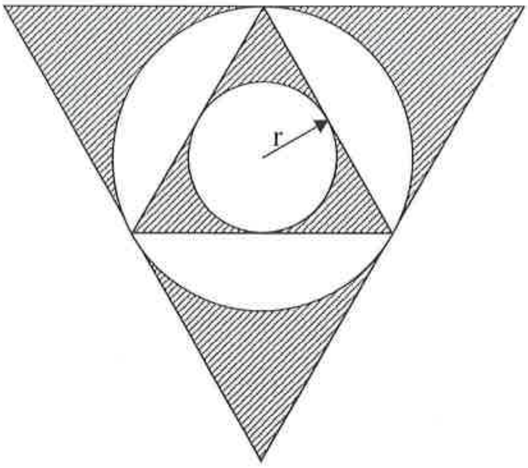
a)



b)



10. In the figure below, all triangles are equilateral. Find the shaded area in terms of r. [7]



For small triangle area = $\left(\frac{3\sqrt{3}}{\text{[illegible]}} - \pi\right) r^2$

[illegible]

For larger area = $4\left(\frac{3\sqrt{3}}{\text{[illegible]}} - \pi\right) r^2$

So

Total area = $5\left(\frac{3\sqrt{3}}{\text{[illegible]}} - \pi\right) r^2$

Part B: Physics [50 Marks]

Multiple choice (10 marks)

Please circle **one** answer to each question only.

11. An ideal transformer has 100 turns on the primary coil. It is connected to an alternating supply of 100 V, 2.4 A. How many turns are required on the secondary coil to supply 4.8 A?

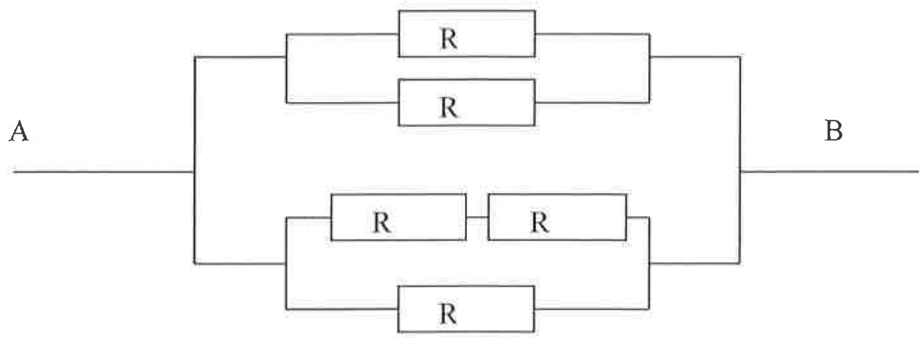
    A 25 turns
    B 50 turns
    C 75 turns
    D 200 turns [2]

12. A radioactive sample contains two different isotopes, A and B. A has a half-life of 3 days, B has a half-life of 6 days. Initially in the sample there are twice as many atoms of A as of B. At what time will the ratio of the number of atoms of A to B be **reversed**?

    A 3 days
    B 6 days
    C 12 days
    D ratio will never be reversed [2]

13. Consider the resistor network shown below. What is the overall resistance between A and B?

A 2 R/7
B R/2
C 3 R/2
D 7 R/2 [2]



14. Two satellites are in orbit around the Earth. The first is in a geostationary orbit, the second satellite orbits at a radius half that of the first. What is the period of the second satellite?

A approx. 4.3 hours
B approx. 8.5 hours
C approx. 17.0 hours
D approx. 72.0 hours [2]

15. You are in a desert and discover a radio mast. 100 m from the mast you measure 20 W of power from the transmitter. If you require a minimum power level of 1 mW, how far can you go away from the mast and still obtain the minimum power? You may assume the transmitter acts like a point source.

A $1/(10\sqrt{2})$ km
B $\sqrt{20}$ km
C $10\sqrt{2}$ km
D 20 km [2]

Written answers (20 marks)

16. A four wheeled car, of mass 1000 kg, rests on the ground. If each tyre is inflated to 2 bar (where 1 bar = 100 kPa), what area of **each** tyre is in contact with the ground? (Assume a uniform distribution of mass across the car). [4]

$F = 10\,000\,N$

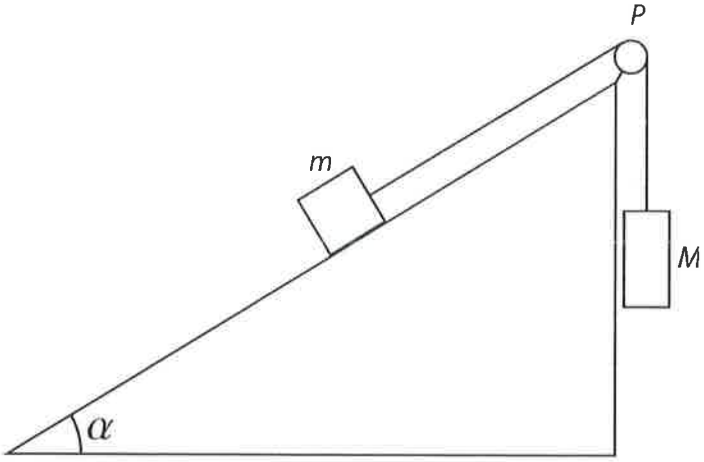
$P = 200\,000\,Pa$

total $A = 0.05\,m^2$

Area of each tyre = $\underline{0.0125\,m^2}$

17. Two masses, m and M, are connected by a massless string of fixed length on a slope inclined at an angle α as sketched in the figure below. The pulley P is massless. Ignoring friction, calculate the acceleration of mass m and the tension of the string. What is the condition for the masses to be stationary?

[5]

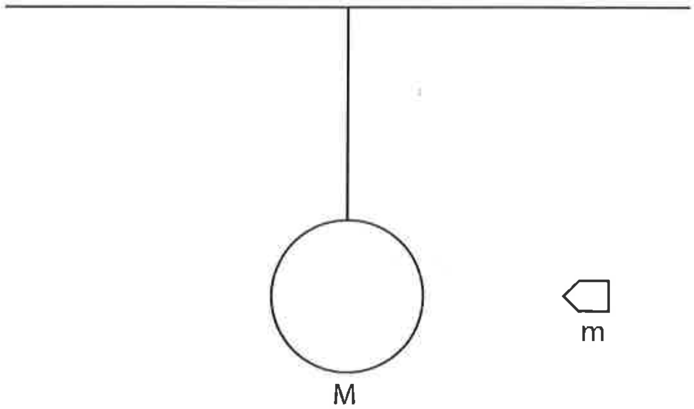


$$a = g\left(\frac{M}{m} - \sin\alpha\right)$$

Stationary when $M = m \sin\alpha$

18. A projectile of mass 0.2 kg and speed 122 $\mathrm{m\,s^{-1}}$ hits a ball of mass 12 kg hanging on a massless string of fixed length, as sketched in the figure below. The projectile was moving at the height of the centre of the ball and after hitting the ball it stops inside the ball, i.e. it becomes stationary with respect to the ball. What is the maximum height that the ball (with the projectile inside) will reach above its original position?

[5]



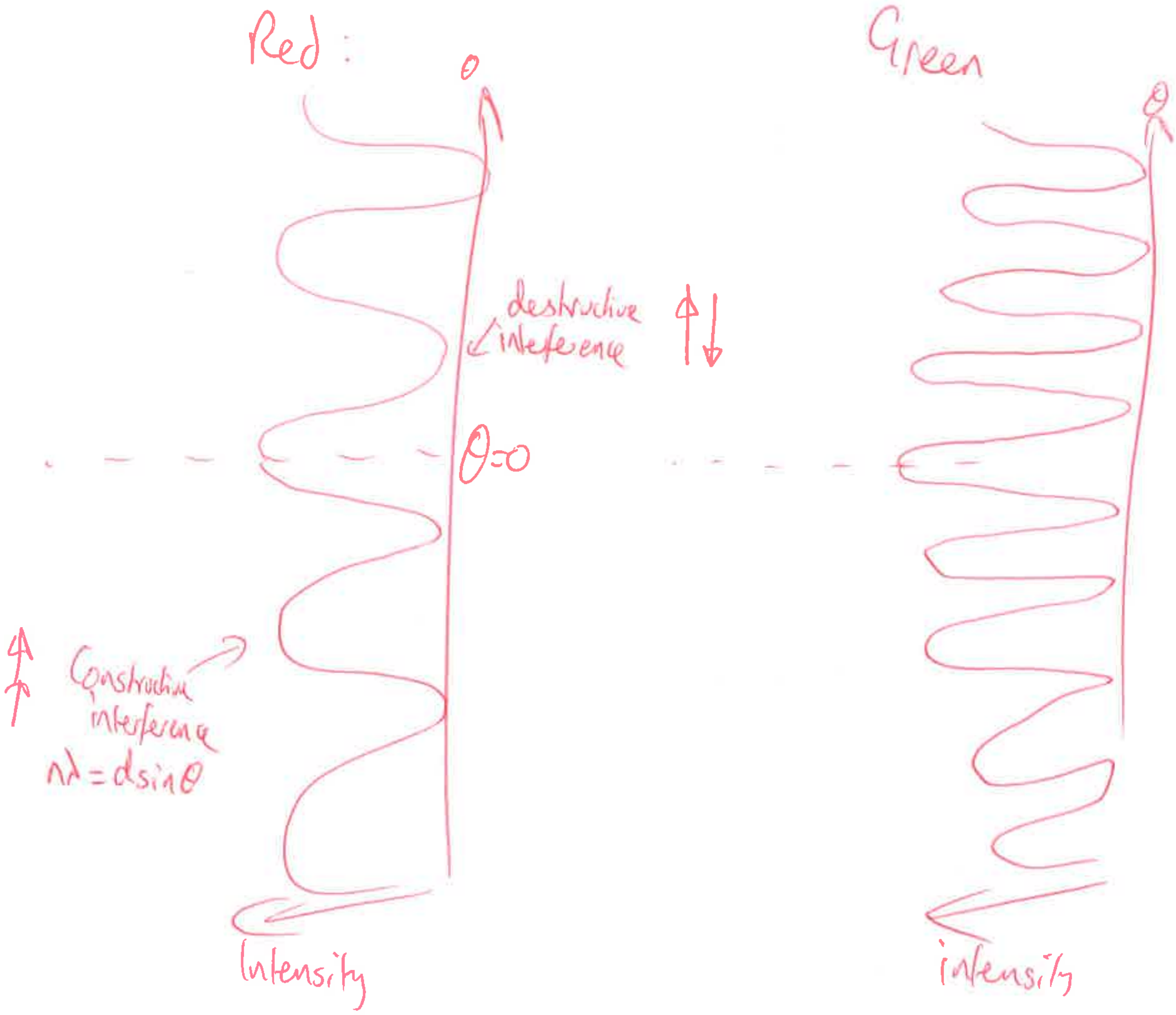
$P_{before} = 122 \times 0.2$

$V_{combined} = \dfrac{122 \times 0.2}{12 + 0.2} = 2\,m/s$

So using $V = \sqrt{2gh}$

$\underline{h = 0.2m}$

19. A monochromatic point light source of wavelength λ is shining through two narrow slits separated by a distance d (d is of the order of λ) on a screen which is a distance D away ($D \gg d$) from the slits. Sketch the pattern of light intensity observed on the screen. Explain why there are minima and maxima. If λ corresponds to red light, what would the pattern look like for green light; make a sketch on the same scale. [6]



Long questions (20 marks)

20. An explorer tests her gas fuelled cooking stove before setting off on an expedition. She has a pot which has a square base of side 10 cm and height 15 cm. Starting from 20 °C, how much energy is required to heat the water in a totally full pot to boiling point? (You may assume the specific heat capacity of water is 4.2 $\mathrm{kJ\,kg^{-1}\,K^{-1}}$ and the density of water is 1 $\mathrm{g\,cm^{-3}}$. You may also neglect the specific heat capacity of the pot.) [4]

$E = mc\Delta\theta$

$= 504\,\mathrm{kJ}$

The explorer now goes up Mount Everest. She discovers that the boiling point of water decreases by 1 °C every 300 m. What physical effect causes this reduction in boiling point? [1]

Lower pressure

When she reaches 6000 m she uses her stove to make a cup of tea. Her mug only requires 100 g of water. How much energy will it take to boil the water and make the cup of tea (assuming it is 10 °C in her tent at 6000 m)? [2]

29.4 kJ

She discovers there is a problem with her stove and it now only produces 50 % of the power it did at sea level. If a full pot took 15 minutes to reach boiling point at sea level, how long will it take to boil the water for the cup of tea? [3]

1.75 mins

21. A particle of mass m moves with a velocity v_0 along the positive x direction before entering a region of length d where it is accelerated by a constant force f acting along the direction x.

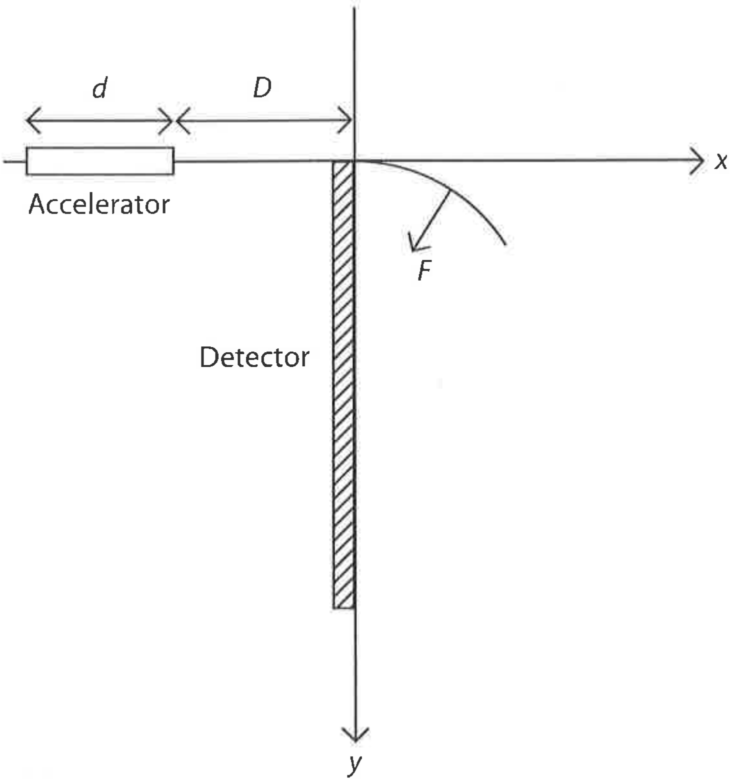
(a) What is the velocity of the particle as it leaves the region of the acceleration?

[1]

$$V^2 = V_0^2 + \frac{2fd}{m}$$

$$V = \sqrt{V_0^2 + \frac{2fd}{m}}$$

After being accelerated, the particle travels a distance D and then enters a region where a force of a magnitude F proportional to its speed v acts on it, $F = \alpha v$ and $\alpha > 0$ is constant. As sketched below, the force acts in the plane of the figure, and is perpendicular to the velocity at every point on the particle trajectory. A detector is placed as shown below, extending downwards, from the point of the entry to the region were the force is acting, along the direction y. (There is no gravitational force involved.)



(b) Derive an expression for the y coordinate where the particle is detected. [4]

y co-ord $= +2r$

where $r =$ radius

$$\frac{mv^2}{r} = \alpha v$$

$$r = \frac{mv}{\alpha}$$

So $y = +\frac{2m}{\alpha}\sqrt{v_0^2 + \frac{2Fd}{m}}$

Now, instead of one particle, there are many particles, initially following the same path as the first particle with speeds v, ranging between v_1 and v_2, at the point of entry to the region where the force F is acting. The detector has a CCD like structure, meaning it is segmented into pixels of size $\triangle y$, the same for all y.

(c) What is the minimal spread of the speeds $\triangle v > 0$ such that v and $v + \triangle v < v_2$ are resolved by the detector? [2]

$$\Delta v = \frac{\alpha \Delta y}{2m}$$

(d) For a given particle, how much work is done by both the forces involved from when the particle enters the accelerator until it strikes the detector?

[3]

Only in acc^{n} so $W = \underline{fd}$