Answers to examination-style questions

### Answers

<p>| | | | |</p>
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<tr>
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<tbody>
<tr>
<td><strong>1 (a)</strong></td>
<td>Taking natural logs on both sides of $V = V_o e^{-t/CR}$ gives $\ln V = \ln V_o + \ln (e^{-t/CR})$</td>
<td>1</td>
<td></td>
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<tr>
<td></td>
<td>As $\ln (e^{-t/CR}) = -\frac{t}{CR}$</td>
<td>1</td>
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<tr>
<td></td>
<td>then $\ln V = \ln V_o - \frac{t}{CR} = a - bt$</td>
<td>1</td>
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<tr>
<td></td>
<td>hence $a = \ln V_o$ and $b = \frac{1}{CR}$</td>
<td>1</td>
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### (b) (i)

<table>
<thead>
<tr>
<th>t/s</th>
<th>210</th>
<th>240</th>
<th>270</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean $\frac{V}{V}$</td>
<td>1.427</td>
<td>1.233</td>
<td>1.033</td>
<td>-0.887</td>
</tr>
<tr>
<td>$\ln V$</td>
<td>0.356</td>
<td>0.209</td>
<td>0.032</td>
<td>-0.120</td>
</tr>
</tbody>
</table>

1 mark for each correct row

### (ii)

![Graph](image)

- For correct labels on each axis
- For suitable scales
- For correctly plotted points
- For best-fit line.

### (iii) Time constant ($= RC$)

\[
\text{Gradient of graph} = 5.40 \times 10^{-3} \text{ s}^{-1}
\]

\[
\therefore \text{time constant} = \frac{1}{5.40 \times 10^{-3}} = 185 \text{ s}
\]

or 190 s

\[
C = \frac{\text{time constant}}{R} = \frac{185}{6.8 \times 10^4} = 2.72 \times 10^{-3} \text{ F}
\]

= 2720 µF or 2700 µF

### (c) (i) The range of each set of readings is no more than 0.03 V except for the reading at $t = 150$ s which is 0.12 V. This exception is probably due to an anomalous reading. The readings are therefore reliable because the range of each set of readings is very small compared with the mean value.

A reliable reading is one that has a consistent value each time the reading is made.
(ii) Apart from the exception at \( t = 150 \) s, the uncertainty of the readings is therefore no more than \( \pm 0.02 \) V.  

\[ \text{or:} \]

At \( t = 0 \) the values of \( \ln V_0 \) are 1.504, 1.506 and 1.504; a mean of 1.505 and an uncertainty of 0.001 (half the range).  
Using the values at 300 s as an example, the values of \( \ln V \) are \( -0.117, -0.105 \) and \( -0.139 \); a mean of \( -0.120 \) and an uncertainty of 0.017 (half the range).  
In finding the gradient the change in \( \ln V \) is divided by the time taken.  
The uncertainty in the change in \( \ln V \) from \( t = 0 \) to 300 s is about 0.017 + 0.001 = 0.018 and as a percentage  
\[ \frac{0.018}{1.504 + 0.120} \times 100 = 1.3\% \]
The equation for \( C \) is  
\[ C = \frac{t}{R(\ln V_0 - \ln V)} \]
The error in \( t \) is not known, but taking the largest value of \( t = 300 \) s, this error is likely to be small (if timing was by hand then the uncertainty in \( t \) will be about 0.3 s or 0.1 %) given \( R \) is accurate to 1%, the value of \( C \) is accurate to within 2.3% (= 1.3% + 1%) or about 2%.  

4 marks max for part (c)  

Note If time permits, you could estimate the random error in \( V \) and hence in \( \ln V \) for every point and represent them as error bars on the graph. This would allow you to draw lines of maximum and minimum gradient and so determine maximum and minimum values for the time constant to give an uncertainty value for \( C \).  

2 (a) Charge on capacitor \( Q = CV \)  
\[ = 2.0 \times 10^{-6} \times 150 \]  
\[ = 3.0 \times 10^{-4} \text{ C (300 } \mu\text{C)} \]

(b) (i) The required graph is of \( pdV \) on the vertical axis and charge \( Q \) on the horizontal axis.  

(ii) Energy stored = \( \frac{1}{2}CV^2 \)  
\[ = \frac{1}{2} \times 2.0 \times 10^{-6} \times 150^2 \]  
\[ = 2.25 \times 10^{-2} \text{ J (22.5 or 23 mJ)} \]  

The upper plate loses electrons and the lower plate gains an equal number of electrons. The charge on the upper plate is \( +300 \mu\text{C} \) and that on the lower plate is \( -300 \mu\text{C} \).  

See Question 1 (above). For the energy stored to be calculated from the area under the line on the graph, the axes have to be this way round. The derivation using \( \Delta W = V \Delta Q \) requires \( \Delta Q \) to be on the horizontal axis.  

You could use \( \frac{1}{2}QV \), where \( Q \) is the 300 \( \mu\text{C} \) determined in part (a). When finding energy stored, \( \frac{1}{2}CV^2 \) is usually the safest approach because both \( Q \) and \( V \) change as a capacitor charges, but \( C \) is constant as \( V \) changes.
3 (a) Charge stored $Q = CV$
$= 330 \times 10^{-6} \times 9.0 = 2.97 \times 10^{-3}$ C
Energy stored $E = \frac{1}{2} Q V$
$= \frac{1}{2} \times 2.97 \times 10^{-3} \times 9.0 = 1.34 \times 10^{-2}$ J

(b) Time constant $= RC$
$= 470 \times 10^{-3} \times 330 \times 10^{-6} = 155$ s

(c) When $t = 60s$, $Q = Q_0 e^{-t/RC}$ gives
$Q = 2.97 \times 10^{-3} e^{-60/155}$
$= 2.02 \times 10^{-3}$ C
pd across capacitor $V = \frac{Q}{C} = \frac{2.02 \times 10^{-3}}{330 \times 10^{-6}}$
$= 6.11$ V

4 (a) Use of $Q = CV$ gives
charge stored $Q = 4.7 \times 10^{-6} \times 6.0$
$= 2.82 \times 10^{-3}$ C or $2.8 \times 10^{-3}$ C

(b) Use of $E = \frac{1}{2} CV^2$
From graph, when $t = 35$ ms, $V = 2.0$ V
$\therefore E = \frac{1}{2} \times 4.7 \times 10^{-6} \times 2.0^2$
$= 9.4 \times 10^{-8}$ J
(c) The time constant is the time taken for \( V \) to decrease from \( V_0 \) to \( (V_0/e) \):
\[ t = \frac{32 \times 10^{-3}}{C} = 32 \times 10^{-3} \]
\[ = 6.81 \times 10^3 \Omega \text{ or } 6.8 \text{k}\Omega \]

Other solutions are possible, but all of them are less direct, for example using
\[ V = V_0 e^{-t/RC} \]
\[ t = 35 \text{ ms, when} \]
\[ V = 2.0 \text{ V, gives } 2.0 = 6.0 e^{-35/RC} \]
\[ \therefore e^{35/RC} = 3.0 \]
\[ \therefore \text{time constant} \]
\[ RC = \frac{35}{\ln 3.0} = 32 \text{ ms} \]

\[ V \text{ must fall to } (6.0/e) = 2.2 \text{ V} \]

From the graph, \( V = 2.2 \text{ V when } t = 32 \text{ ms} \)
\[ \therefore \text{time constant} \]
\[ RC = \frac{35}{\ln 3.0} = 32 \text{ ms} \]

(The graph shows a value between 32 and 33 ms)

(d) Time constant = \( RC \)
\[ \therefore \text{resistance} \]
\[ R = \frac{32 \times 10^{-3}}{C} = 32 \times 10^{-3} \]
\[ = 6.81 \times 10^3 \Omega \text{ or } 6.8 \text{k}\Omega \]

This question shows how you might find the resistance of a resistor from a data
logging experiment, using the discharge of a capacitor of known capacitance.

5 (a) Time constant = \( RC \)
\[ \therefore \text{capacitance of} \]
\[ C = \frac{2.2 \times 10^{-4}}{R} \]
\[ = \frac{2.2 \times 10^{-4}}{220} = 1.0 \times 10^{-6} \text{ F (1.0 } \mu \text{F)} \]

This circuit needs to have a very small
time constant, so that it can be assumed
that the capacitor discharges fully whilst
the switch is touching contact \( Y \). The
farad is a very large unit, so practical
capacitors usually have their values
marked in \( \mu \text{F}, \text{nF or pF}. \)

(b) Periodic time of oscillation of switch
\[ T = \frac{1}{f} = \frac{1}{400} \]
\[ = 2.5 \times 10^{-3} \text{ s (2.5 ms)} \]

The question states that the switch
touches each contact (\( X \) and \( Y \)) 400 times
per second. It therefore oscillates at a
frequency of 400 Hz.

(c) (i) Switch makes contact with \( Y \) for a
time \( t = \frac{T}{2} = 1.25 \times 10^{-3} \text{ s (1.25 ms)} \)
and the time constant of the circuit is
0.22 ms
\[ V = V_0 e^{t/RC} \]
gives \( V = 12 e^{-1.25/0.22} \)
\[ \therefore \text{pd across capacitor at time } T/2 \]
\[ V = 4.09 \times 10^{-2} \text{ V (40.9 or 41 mV)} \]

(ii) It is reasonable to assume that the capacitor has completely discharged in
time \( T/2 \) because either
- 40.9 mV is only 0.34% of 12 V, or
- 1.25 ms is almost 6 time constants

6 (a) A capacitor has a capacitance of 1 farad if it stores 1 coulomb of charge when the pd across it is 1 V
\[ C = \frac{Q}{V} \]
\[ \therefore 1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}} \]
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| 4.14 \times 10^{-10} J | 1 | Similarly: 
| 1.38 \times 10^{-10} C or 1.4 \times 10^{-10} C | 1 | Hence: 
| \frac{1}{2} \times 2.3 \times 10^{-11} \times 6.0^2 | 1 | Alternatively: 
| 2.3 \times 10^{-11} \times 6.0 | 1 | E = \frac{1}{2} QV = \frac{1}{2} \times 1.38 \times 10^{-10} \times 6.0 |
| 10^{-10} J | 1 | \therefore e^{-0.0004/RC} = 6.0/80 = 7.5 |
| 6.0 \times 2.3 \times 10^{-11} \times 8.63 \times 10^{6} \Omega | 1 |Taking logs, \frac{0.0004}{RC} = \ln 7.5 = 2.01 |
| \frac{1}{2} \times 2.3 \times 10^{-11} | 1 | Giving \( RC = 1.99 \times 10^{-4} \) s |
| 8.36 \times 10^{6} \Omega | 1 | Resistance of oscilloscope |
| 1.99 \times 10^{-4} | 1 | \therefore e^{0.0004/RC} = 6.0/80 |
| 2.3 \times 10^{-11} | 1 | Taking logs, \frac{0.0004}{RC} = \ln 7.5 = 2.01 |
| 8.6 \times 10^{6} \Omega | 1 | Giving \( RC = 1.99 \times 10^{-4} \) s |
| 1.22 \times 10^{-3} \Omega or 1.2 \times 10^{-3} J | 1 | Resistance of oscilloscope |
| 2.2 \times 10^{-3} | 1 | When a capacitor is charged from a battery of emf \( V \) so that it stores charge \( Q \), the battery moves a total charge \( Q \) across a pd of \( V \) and does work \( QV \). Half of this work is lost as thermal energy as the charge flows through resistive components in the circuit. |
| 2.1215 \times 10^{-3} | 1 | Alternatively: 
| 2.43 \times 10^{-3} J | 1 | \( E = \frac{1}{2} CV^2 \) means that \( E \propto V^2 \). In this case, \( V \) decreases to 0.1 (or 10\%) of its initial value, so \( E \) decreases to 0.01 (or 1\%) of its initial value. Therefore 99\% of the initial energy is released. |
| 2 \times 1.215 \times 10^{-3} = 2.43 \times 10^{-3} J | 1 | The torch bulb gives out light from \( t = 0 \) (when \( V = 3.0 \) V) until the time when \( V \) has fallen to 2.0 V. This time is calculated by solving the exponential decay equation, involving the use of logs once more. |
### Chapter 6

#### Answers to examination-style questions

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| (iii) Energy of one photon  
\[ \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{500 \times 10^{-9}} \]  
\[ = 3.98 \times 10^{-19} \text{ J} \]  

Energy released when capacitor discharges from 3.0 V to 2.0 V  
\[ = 6.80 \times 10^{-4} \text{ J} \]  

Number of photons released  
\[ = \frac{6.80 \times 10^{-4}}{3.98 \times 10^{-19}} = 1.71 \times 10^{15} \]  
or \[ 1.7 \times 10^{15} \]  

The final part of the question tests whether you can remember how to calculate the energy of a photon (covered in AS Physics Unit 1).  
A wavelength of 500 nm corresponds to the average wavelength of visible light. The conclusion of the calculation is that the tiny amount of energy released as the capacitor discharges will produce an incredibly large number of photons.  

8 (a) Time constant of circuit \( RC \)  
\[ = 680 \times 2.2 \times 10^{-6} = 1.50 \times 10^{-3} \text{ s} \]  

Use of \( V = V_0 e^{-t/RC} \) gives \( 2.2 = 5.0 e^{-t/RC} \)  

from which \( t = RC \ln \frac{5.0}{2.2} \)  

\[ = 1.50 \times 10^{-3} \times 0.821 \]  

\[ \therefore \text{time of contact } t = 1.23 \times 10^{-3} \text{ s (1.23 ms)} \]  

This question is an example of the practical use of a capacitor–resistor discharge circuit to measure a very short time. The time for which the metal ball is in contact with the metal block would be much too short to be measured directly. Charge flows from the capacitor and through the resistor whilst the circuit is complete: this only happens during the time when the ball and block make contact.  

(b) (i) Initial energy stored \( E_1 = \frac{1}{2} CV^2 \)  
\[ = \frac{1}{2} \times 2.2 \times 10^{-6} \times 5.0^2 \]  
\[ = 2.75 \times 10^{-6} \text{ J} \]  

Final energy stored  
\[ E_2 = \frac{1}{2} \times 2.2 \times 10^{-6} \times 2.2^2 \]  
\[ = 5.32 \times 10^{-6} \text{ J} \]  

energy lost by capacitor \( = E_1 - E_2 \)  
\[ = 2.22 \times 10^{-5} \text{ J} \]  

The steps in this calculation could be combined into one expression:  
\[ \Delta E = E_1 - E_2 = \frac{1}{2}C(V_1^2 - V_2^2) \]  
\[ = \frac{1}{2} \times 2.2 \times 10^{-6} \times (5.0^2 - 2.2^2) \]  
\[ = 2.22 \times 10^{-5} \text{ J} \]  

Whenever charge flows there is a current. A current in a resistor causes its internal energy (and therefore its temperature) to increase. When its temperature is higher than the surroundings, the resistor passes energy away, raising the internal energy of the air around it.  

The energy is dissipated in the 680 \( \Omega \) resistor.  
This energy becomes internal energy of the resistor, and eventually internal (thermal) energy in the surroundings.  

Nelson Thornes is responsible for the solution(s) given and they may not constitute the only possible solution(s).