Ideal Gases

- ***Candidates should be able to*** :

  - ***State Boyle's law.***
  - ***Select*** *and* ***apply*** : $\frac{pV}{T} = \text{constant}$
  - ***State*** *the basic assumptions of the* ***kinetic theory of gases.***
  - ***State*** *that* ***1 mole*** *of any substance contains* 6.02×10^{23} ***particles*** *and that* 6.02×10^{23} mol^{-1} *is the* ***Avogadro constant,*** N_A.
  - ***Select*** *and* ***solve*** *problems using the* ***Ideal gas equation*** *expressed as :*

$$pV = nRT \quad \text{and} \quad pV = NkT$$

Where N *is the* ***number of atoms*** *and* n *is the* ***number of moles.***

  - ***Explain*** *that the* ***mean translational kinetic energy*** *of an atom of an ideal gas is* ***directly proportional to the absolute gas temperature in Kelvin.***
  - ***Select*** *and* ***apply*** *the equation :* $E = 3/2\, kT$

For the ***mean translational kinetic energy*** *of atoms.*

THE GAS LAWS

BOYLE'S LAW

This law, first discovered by ***Robert Boyle*** *in 1662, relates the* ***PRESSURE*** *and* ***VOLUME*** *of a gas at* ***CONSTANT TEMPERATURE*** *and states that :*

The PRESSURE (p) of a fixed mass of gas at constant temperature is *inversely proportional* to its VOLUME (V).

Stated mathematically :

$$p \propto 1/V$$

OR

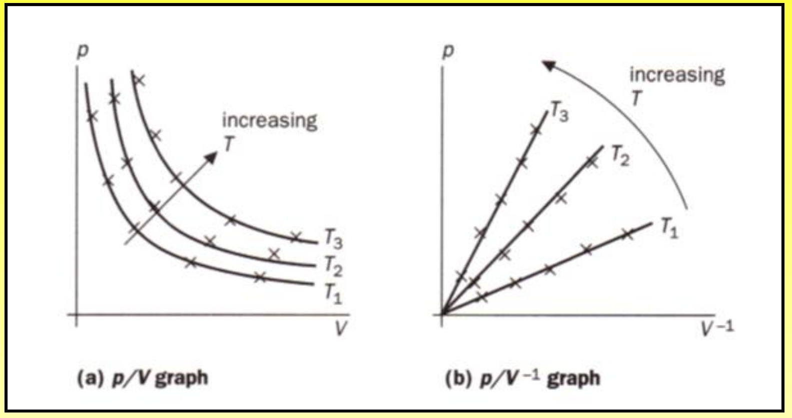
$$pV = \text{constant}$$

- *This equation can be used to calculate* ***pressure*** *or* ***volume*** *changes whenever a fixed mass of gas is either* ***compressed*** *into a smaller volume (at higher pressure) or allowed to* ***expand*** *into a larger volume (by reducing the pressure), providing the* ***temperature remains the same*** *throughout the change.*

- *So if* (p_1) *and* (V_1) *are the* ***pressure*** *and* ***volume*** *of a fixed mass of gas at some initial stage and* (p_2) *and* (V_2) *are the values after expansion or compression* ***at constant temperature.*** *Then :*

$$p_1V_1 = p_2V_2$$

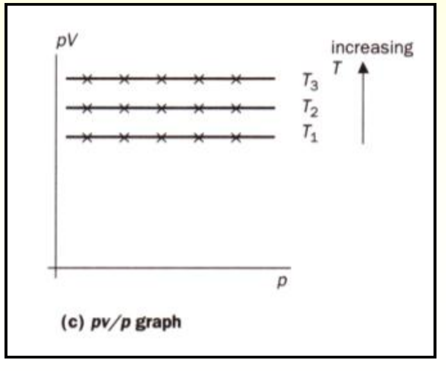
- *A graphical representation of the relation between the **pressure (p)** and the **volume (V)** of a fixed mass of gas **at constant temperature** is shown below.*



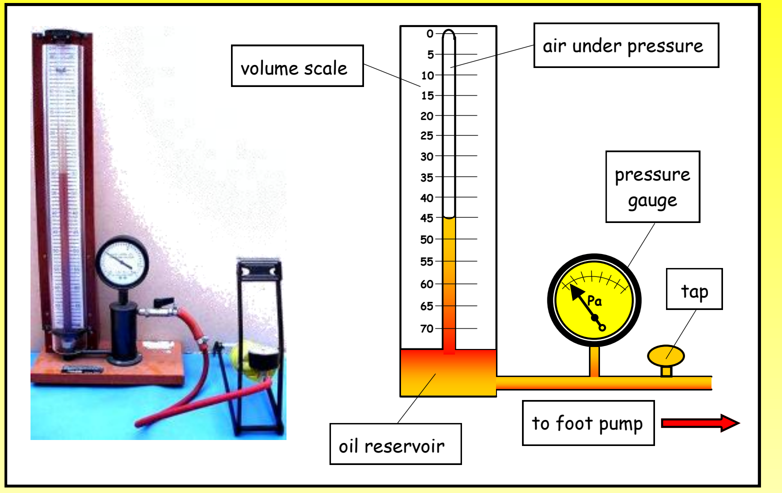
- *When **p** is plotted against **V**, a curve called a **rectangular hyperbola** is obtained and when **p** is plotted against **1/V**, the result is a **straight-line** graph.*

  *By investigating a fixed mass of gas at **different temperatures**, a series of graphs is obtained. Each of the curves or lines is called an **ISOTHERMAL** (since the plotted values are all for the same temperature).*

- *Plotting **pV** against **p** Yields horizontal lines for each different temperature as shown in the diagram opposite.*



PRACTICAL INVESTIGATION OF BOYLE'S LAW



- The apparatus shown above may be used to investigate the relation between the **pressure** and **volume** of a fixed mass of gas **at constant temperature.**

- A long glass tube which is closed at one end and mounted against a volume scale, contains the fixed mass of air under test. The pressure on the air column can be varied using a foot pump which forces oil from the reservoir up the tube and so compresses the air above it. The pressure gauge measures the pressure in pascal and the volume of the air column is read directly from the scale beneath the tube.

- The pump is first used to compress the air to its smallest possible volume and the tap is closed. In order to ensure that the oil level has stabilised and that the air is at room temperature, the **pressure (p)** and **volume (V)** readings are not taken for a couple of minutes.
  The apparatus is then slowly vented by opening the tap slightly and then closing it again. Once again the corresponding pressure and volume readings are taken after a short time has elapsed. This process is repeated several times so as to obtain a set of corresponding **p** and **V** values.

RESULTS

PRESSURE, p/x 10^5 Pa	VOLUME, V/cm^3	1/V/cm^{-3}	pV/Nm

CALCULATIONS AND ANALYSIS

- Plot a graph of **p against 1/V**. Is it a best-fit straight line ? If so, what does this tell you about the relationship between p and 1/V ?
- Convert the volume readings to **m^3** by multiplying by **10^{-6}** and then calculate the product **pV** for each set of corresponding **p** and **V** values. Is the product **pV approximately constant** throughout ?
- Has **Boyle's law** been verified by the results of the experiment ?

CHARLES' LAW

*This law relates the **VOLUME (V)** and the **TEMPERATURE (T)** of a gas at **CONSTANT PRESSURE** and states that :*

The VOLUME (V) of a fixed mass of gas at constant pressure is *directly proportional* to its ASOLUTE (KELVIN) TEMPERATURE (T).

Stated mathematically :

$$V \propto T$$

OR

$$\frac{V}{T} = \text{constant}$$

From which :

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

PRESSURE LAW

*This law relates the **PRESSURE (p)** and the **TEMPERATURE (T)** of a gas at **CONSTANT VOLUME** and states that :*

The PRESSURE (p) of a fixed mass of gas at constant volume is *directly proportional* to its ABSOLUTE (KELVIN) TEMPERATURE (T).

Stated mathematically :

$$p \propto T$$

OR

$$\frac{p}{T} = \text{constant}$$

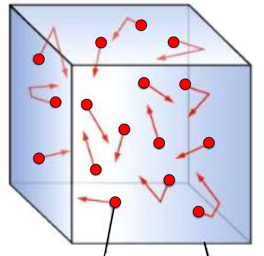
From which :

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

ASSUMPTIONS OF THE KINETIC THEORY OF GASES

- *When we talk about the large-scale properties of a gas, such as **MASS, VOLUME, TEMPERATURE** and **PRESSURE**, we are dealing with a **MACROSCOPIC** description of the gas.*

- *The **MICROSCOPIC** description involves a gas model consisting of a large number of identical molecules in rapid, random motion, colliding with each other and with the walls of the containing vessel.*



- *The **KINETIC THEORY OF GASES** relates the **MICROSCOPIC** and **MACROSCOPIC** properties of a gas and is governed by the following basic assumptions :*

  - **A gas consists of a large number of identical molecules in a state of rapid, random motion.**
  - **The collisions between molecules and the walls of the containing vessel are ELASTIC.**
  - **Other than those which occur during collisions, the forces between the molecules is zero.**
  - **The gravitational force on the molecules is negligible.**
  - **Compared to the volume of the containing vessel, the total volume of the molecules is negligible.**

TERMS RELATING TO THE MICROSCOPIC MODEL OF GASES

AVOGADRO'S HYPOTHESIS *states that **equal volumes** of gases at the **same temperature and pressure** contain **equal numbers of molecules.***

The **AVOGADRO CONSTANT or NUMBER** (N_A) is the number of atoms in **12 g of carbon-12.**

N_A is the number of atoms per **MOLE*** of gas = **6.02 x 10^{23}**

* 1 MOLE is the amount of a substance that contains 6.02 x 10^{23} particles (i.e. atoms or molecules).

The **MOLARITY** (in mol) of a substance is the **NUMBER OF MOLES** contained in a certain amount of the substance.

The **MOLAR MASS (M_m) (in kg mol^{-1} or g mol^{-1})** of a substance is the **MASS PER MOLE** of the substance.
(e.g. **M_m** of oxygen gas = **0.032 kg mol^{-1} or 32 g mol^{-1}**).

To calculate the **NUMBER OF MOLES (n)** in **MASS(M)** of gas of **MOLAR MASS (M_m)** we use :

$$n = M/M_m$$

To calculate the **NUMBER OF MOLECULES (N)** in **n MOLES** of gas we use :

$$N = n N_A = \frac{M}{M_m} N_A$$

THE IDEAL GAS EQUATION

- *An **IDEAL GAS** is one which obeys the **GAS LAWS** exactly and for which is subject to the assumptions of the **KINETIC THEORY OF GASES**.*

- *So for a ideal gas each of the following applies :*

  - ***pV = constant*** *(at constant temperature)*
  - ***V/T = constant*** *(at constant pressure)*
  - ***p/T = constant*** *(at constant volume)*

From which we have that :

$$\frac{pV}{T} = \text{constant}$$

The magnitude of the constant depends on the mass of gas being considered.

*For **1 MOLE** of gas, the constant is the **UNIVERSAL MOLAR GAS CONSTANT (R) = 8.31 J mol^{-1} K^{-1}**.*

*So for **1 MOLE** of gas :* $\frac{PV}{T} = R$

So : $pV = RT$

*For **n MOLES** of gas :*

$$pV = nRT$$

(Pa) (mol) (K)

(m^3) (J mol^{-1} K^{-1})

*This is known as **THE IDEAL GAS EQUATION**.*

- *The ideal gas equation may also be expressed in terms of the **BOLTZMANN CONSTANT (k)** which is **the gas constant per molecule (i.e. $k = R/N_A$).***

$$k = 1.38 \times 10^{-23} \text{ J kg}^{-1}$$

Since $k = R/N_A$, *then* $R = kN_A$

Substituting for R in $pV = nRT$ *gives* $pV = n(kN_A)T$

But nN_A = ***number of moles of gas x number of molecules per mole***

= ***number of molecules in the gas (N)***

$$pV = NkT$$

(Pa) (m^3) (K)

(J kg^{-1})

- *This form of the ideal gas equation is useful for problems set in terms of molecules rather than moles of gas.*

THE COMBINED GAS EQUATION

- *For **(n) moles** of a gas of **volume** (V_1) at a **pressure** (p_1) and **temperature** (T_1) :*

$$\frac{p_1V_1}{T_1} = nR \quad \text{.............(1)}$$

- *For the same amount of gas **(n moles)** whose **volume** has changed to **(V_2)** at a new **pressure** (p_2) and **temperature** (T_2) :*

$$\frac{p_2V_2}{T_2} = nR \quad \text{..............(2)}$$

From (1) and (2) we have that :

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

*This is called the **COMBINED GAS EQUATION.***

- *It is particularly useful for solving problems in which volume, pressure and temperature vary simultaneously.*

- *It does not matter what units are used for **p** and **V**, so long as they are **the same on both sides of the equation**, but **T MUST BE IN KELVINS (K).***

1 (a) A fixed mass of gas has a volume of **3000 cm^3** at a pressure of **1.0 x 10^5 Pa**. Calculate its **volume** when the pressure is increased to **2.5 x 10^6 Pa** with the **temperature remaining constant**.

(b) A fixed mass of gas has a volume **V** when the temperature is **127°C**. To what **temperature** must the gas be raised so that its volume increases to **2.75 V** with the **pressure remaining constant ?**

(c) A fixed mass of gas has a volume of **0.02 m^3** at a pressure of **2.02 x 10^5 Pa** and a temperature of **44°C**. Calculate the new **volume** of the gas at standard temperature and pressure (i.e. **0°C** And **1.01 x 10^5 Pa**).

2 A diver swims at a depth of **40 m** where the temperature of the water is **4.0 °C**. He inhales **1.2 x 10^{-5} m^3** of compressed air at a pressure of **7.0 x 10^5 Pa** and suddenly sees something that panics him into rising to the surface very rapidly without exhaling.

Calculate the **new volume** of the air which he inhaled at **40 m**, if the surface temperature and pressure is **20.0 °C** and **1.01 x10^5 Pa** respectively.

3 (a) How many **moles** are there in **1.6 kg** of oxygen if the molar mass of this gas is **32 g mol^{-1} ?**

(b) Calculate the **volume** occupied by **1 mole** of an ideal gas at a temperature of **0°C** and a pressure of **1.013 x10^5 Pa**.

(The universal molar gas constant, ***R*** = **8.31 J mol^{-1} K^{-1}**).

4 The molar mass of nitrogen is **0.028 kg mol^{-1}**. A sample of the gas contains **6.02 x 10^{22}** molecules.

(a) Calculate :

(i) The **number of moles** of gas contained in the sample.

(ii) The **mass** of the gas sample.

(iii) The **volume** occupied by the gas at a pressure of **1.01 x 10^{5} Pa** and a temperature of **17°C**.
(The Avogadro number, **N_A = 6.02 x 10^{23} mol^{-1}**).

(b) A sample of a different gas contains **1.204 x 10^{25} molecules.** Calculate the **volume** occupied by this gas sample at a pressure of **2.02 x 10^{6} Pa** and a temperature of **100°C.**
(The Boltzmann constant, **k = 1.38 x 10^{-28} J kg^{-1}**).

5 (a) Sketch a graph to show how the **pressure** of **2 moles** of gas varies with **temperature** when the gas is heated from **20°C** to **100°C** in a sealed container of volume **0.050 m^{3}**.

(b) If the molar mass of the gas in (a) is **0.032 kg mol^{-1}**, calculate the **density** of the gas.

6 A vehicle air bag inflates rapidly when an impact causes the production and release of a large quantity of nitrogen in a chemical reaction. In a test of an air bag, the bag inflates to a volume of **1.2 m^{3}** and a pressure of 103 kPa at a final temperature of **280 K.**

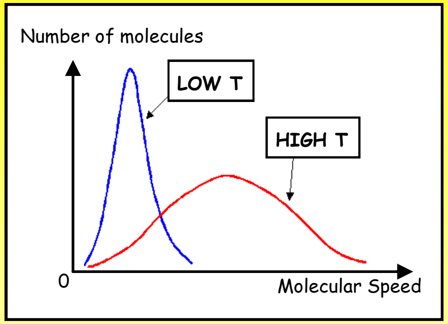
Calculate : (a) The **number of moles** of gas in the bag.

(b) The **initial pressure** of the gas if it was released from a container of volume **5.6 x 10^{-4} m^{3}** at the **same temperature.**

SPEED OF GAS MOLECULES

The molecules in a gas have a wide range of different speeds.

This is illustrated In the graph shown opposite In which ***NUMBER OF MOLECULES*** *is plotted against* ***MOLECULAR SPEED.***



From the graph it can be seen that :

- **Few** molecules have either **very low** or **very high** speed and **none** have **zero speed** (i.e. are stationary).

- The distribution of gas molecular speed depends on **temperature**. As the gas temperature **increases,** the distribution curve becomes **flatter and broader**.

  The **higher** the temperature, the **greater** is the proportion of **high-speed** molecules and the **smaller** is the proportion of **low-speed** molecules.

The ***MEAN SPEED ($\bar{c}$)*** *is the average value of the speeds of* ***all*** *the molecules.*

MOLECULAR KINETIC ENERGY AND TEMPERATURE

Since the mean speed ($\bar{c}$) of the molecules in a gas increases with temperature, the mean translational kinetic energy of the molecules must also increase with temperature.

The derivation which follows will establish that :

The *mean translational kinetic energy* of a molecule of an ideal gas is directly proportional to the *absolute temperature* of the gas.

*The equation of state for **(n) moles** of an ideal gas of **volume (V)** at a **pressure (p)** and an **absolute temperature (T)** is :*

$$pV = nRT \quad \ldots\ldots (1)$$

*Using **kinetic theory of gases**, if the gas is made up of **(N) molecules** each of **mass (m)** and moving with a **mean speed ($\bar{c}$)** :*

$$pV = \tfrac{1}{3} Nm\bar{c}^2 \quad \ldots\ldots (2)$$

Combining (1) and (2) gives :

$$\tfrac{1}{3} Nm\bar{c}^2 = nRT$$

Which may be expressed as :

$$\tfrac{2}{3} N(\tfrac{1}{2}m\bar{c}^2) = nRT$$

Therefore :

$$\tfrac{1}{2}m\bar{c}^2 = \frac{3n}{2N}RT = \frac{3}{2}\frac{R}{N/n}T$$

*But **N/n = The number of gas molecules per mole = N_A**.*

AVOGADRO NUMBER

$$\tfrac{1}{2}m\bar{c}^2 = \frac{3}{2}\frac{R}{N_A}T \quad \ldots\ldots (3)$$

- ***R** is the gas constant **per mole** of gas molecules.*
- ***R/N_A** = The gas constant **per molecule**. = **k***

BOLTZMANN CONSTANT

$$E = 3/2\ kT$$

(J) (J kg^{-1}) (K)

The *mean translational kinetic energy (E)* of a molecule of an ideal gas is directly proportional to the *absolute temperature (T)* of the gas.

NOTE

- From equation (3) : $\bar{c}^2 \propto T$

  i.e. The **mean-square speed of a gas molecule** is directly proportional to the **absolute temperature**.

- Since the internal energy of an **ideal gas** is regarded as being **purely kinetic** :

The sum of the mean kinetic energies of all the molecules in a gas is the internal energy of the gas.

UNIT G484	Module 2	4.3.4	Ideal Gases

- **PRACTICE QUESTIONS (2)**

1 Given that the molar mass of nitrogen is **0.028 kg mol^{-1}** and that the Avogadro number **(N_A)** is **6.02 x 10^{23} mol^{-1}**, calculate :

(a) The **mass** of a molecule of nitrogen.

(b) The **kinetic energy** of a nitrogen molecule which is moving with a speed of **1500 m s^{-1}**.

2 The mean translational kinetic energy of an oxygen molecule at **20°C** is found to be **7.5 x 10^{-21} J**. Calculate :

(a) The **mean speed** of an oxygen molecule.

(b) The **internal energy** of **2.5 moles** of oxygen gas at 20°C.

(c) The **mean translational kinetic energy** of an oxygen molecule at a temperature of **200°C.**

- Molar mass of oxygen = 0.032 kg mol^{-1}.
- Avogadro number, **N_A = 6.02 x 10^{23} mol^{-1}**

3 A cylinder of volume **0.25 m^3** contains nitrogen gas at a pressure of **1.5 x 10^5 Pa** and a temperature of **17°C.** Assuming that the gas behaves **ideally,** calculate :

(a) The **number of moles** of gas in the cylinder.

(b) The **mean translational kinetic energy** of a nitrogen molecule.

(c) The **total kinetic energy** (i.e. the internal energy) of the gas contained in the cylinder.

- Molar mass of nitrogen **M_m = 0.028 kg mol^{-1}.**
- Molar gas constant, **R = 8.31 J mol^{-1} K^{-1}.**
- Avogadro constant, **N_A = 6.02 x 10^{23} mol^{-1}**

- **HOMEWORK QUESTIONS**

1 (a) State **Boyle's law :** (i) As a **written statement.**

(ii) Expressed as a **mathematical equation.**

(iii) Expressed in the form of a **graph relating the quantities involved.**

(b) **State** the main assumptions of the **kinetic theory of gases.**

(c) (i) **Explain** what is meant by an **ideal gas.**

(ii) Write down the **ideal gas equation for (n) moles of gas** and **state** what each quantity in the equation is as well as giving its **unit.**

(d) (i) **Define** the **mole.** How is the mole related to the **Avogadro constant (N_A) ?**

(ii) What is the **molar mass** of a substance ?

(iii) How many moles are there in **(M) kg** of a substance of molar mass **(M_m) kg mol^{-1}?**

(e) How is the **mean translational kinetic energy** of a molecule of an ideal gas related to the gas **temperature.** Give a **written statement** and a relevant **equation** for this.

Ideal Gases

2 (a) The equation of state of an ideal gas is **$pV = nRT$**. **Explain** why the temperature must be measured in **Kelvin**.

(b) A meteorological balloon rises through the atmosphere until it expands to a volume of **1.0×10^6 m^3**, where the pressure is **1.0×10^3 Pa**. The temperature also falls from **17°C to –43°C**.

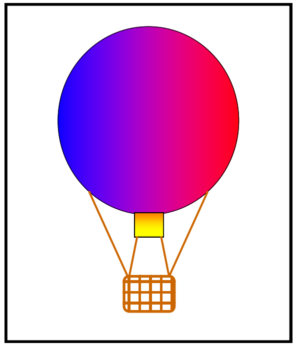
- The pressure of the atmosphere at the Earth's surface = **1.0×10^5 Pa**.

Show that the volume of the balloon at take off is about **1.3×10^4 m^3**.

(c) The balloon is filled with helium gas of molar mass **4.0×10^{-3} kg mol^{-1}** at **17°C** at a pressure of **1.0×10^5 Pa**.
Calculate : (i) The number of moles of gas in the balloon.
(ii) The mass of gas in the balloon.

(d) The internal energy of the helium gas is equal to the random kinetic energy of all of its molecules. When the balloon is filled at ground level at a temperature of **17°C** the internal energy is **1900 MJ**. Estimate the **internal energy** of the helium when the balloon has risen to a height where the temperature is **–43°C**.

(e) The upward force on the balloon at the Earth's surface is **1.3×10^5 N**. The initial acceleration of the balloon is **27 m s^{-2}** and its total mass is **M**.



(i) On the diagram opposite **draw and label arrows** to represent the forces acting on the balloon immediately after take off.

(ii) Calculate the value of **M**.

(OCR A2 Physics - Module 2824 - June 2005)

3 (a) Explain what is meant by the **internal energy** of a gas. *10*

(b) A bicycle tyre has a volume of **2.1×10^{-3} m^3**. On a day when the temperature is **15°C** the pressure of the air in the tyre is **280 kPa**. Assume that the air behaves as an ideal gas.

(i) Calculate the **number of moles (n)** of air in the tyre.

(ii) The bicycle is ridden vigorously so that the tyres warm up. The pressure in the tyre rises to **290 kPa**. Calculate the **new temperature** of the air in the tyre. Assume that no air has leaked from the tyre and that the volume is constant.

(iii) Calculate, for the air in the tyre, the ratio :

$$\frac{\textbf{Internal energy at the higher temperature}}{\textbf{Internal energy at 15°C}}$$

(OCR A2 Physics - Module 2824 - January 2007)

4 A light bulb contains **6.0×10^{-5} m^3** of the inert gas, argon. The gas pressure in the bulb is **16 kPa** when the bulb is unlit and the gas temperature is **20°C**. The molar mass of argon is **0.040 kg mol^{-1}** and the molar gas constant **R** is **8.31 J mol^{-1} K^{-1}**.

(a) (i) Calculate the **number of moles** of argon gas in the bulb.

(ii) Calculate the **number of argon atoms** in the bulb. Argon consists of single atoms that do not combine with each other.

(iii) Calculate the **mean speed** of an argon atom if its kinetic energy at **20°C** is **5.5×10^{-21} J**.

(b) The temperature of the gas in the bulb increases to a maximum of **120°C** once it has been lit for some time. Calculate :

(i) The **gas pressure** at this new temperature.

(ii) The **new mean translational kinetic energy** of an argon atom at this temperature.