- ***Candidates should be able to*** :

  - *Describe simple examples of* **free oscillations.**

  - *Define and use the terms* **displacement, amplitude, period, frequency, angular frequency and phase difference.**

  - **Select** *and* **use** *the equation :* **Period = 1/frequency**

  - *Define* **simple harmonic motion.**

  - **Select** *and* **apply** *the equation : as the defining equation of* **simple harmonic motion.** $a = -(2\pi f)^2 x$

  - **Select** *and* **use :** $x = A\cos(2\pi ft)$ & $x = A\sin(2\pi ft)$

    as solutions of the equation : $a = -(2\pi f)^2 x$

  - **Select** *and* **apply** *the equation : for the* **maximum speed of a simple harmonic oscillator.** $v_{max} = (2\pi f)A$

  - **Explain** *that the* **period** *of an object with simple harmonic motion, is* **independent of its amplitude.**

  - **Describe with graphical illustrations,** *the changes in* **Displacement, velocity** *and* **acceleration** *during simple harmonic motion.*

  - **Describe** *and* **explain** *the interchange between* **kinetic** *and* **potential energy** *during simple harmonic motion.*

  - **Describe** *the effects of* **damping** *on an oscillatory system.*

  - **Describe** *practical examples of* **forced oscillations** *and* **resonance.**

  - **Describe graphically** *how the* **amplitude** *of a forced oscillation changes with* **frequency** *near to the* **natural frequency** *of the system.*

  - **Describe** *examples where resonance is* **useful** *and other examples where resonance* **should be avoided.**

- ***FREE AND FORCED OSCILLATIONS***

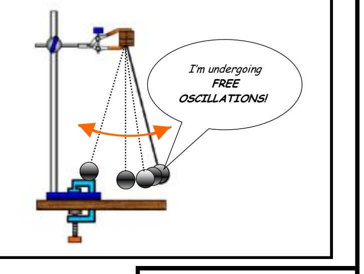
- *An object is said to* **OSCILLATE** *or* **VIBRATE** *when it moves back and forth repeatedly on either side of some fixed position (called the* **EQUILIBRIUM POSITION***), to which it returns when the oscillation ceases.*

  There are many examples of oscillating objects, ranging from the thermal vibrations of atoms in a solid to the swaying motion of the top of a skyscraper in a strong wind.

  Some oscillations, like those produced in a plucked guitar string or the skin of a banged drum, only occur for a short time period. Others, like the beating of a humming bird's wings, are so fast that we are unable to follow them with the naked eye. And yet others, such as x-rays, microwaves and radio waves, are beyond the reach of our senses.

- *In the case of* **FREE OSCILLATIONS** *there is no driving mechanism and the oscillating object continues to move for some time after it has initially been set into oscillation.*

  A good example of a system undergoing free oscillations is that of a pendulum which is slightly displaced from its central equilibrium position and then released.

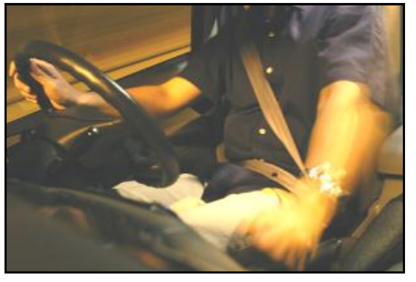


When an object is set into **FREE OSCILLATION**, *it will vibrate at a particular frequency, called the* **NATURAL FREQUENCY** *of vibration.*

The **NATURAL FREQUENCY** *of vibration of an oscillator is that frequency with which it will vibrate freely after an initial disturbance.*

- *A* **FORCED OSCILLATION** *occurs when an object is caused to vibrate by a periodic driving force. This makes the object vibrate at the forcing frequency.*

A good example of this is when engine vibrations are felt in the steering wheel and gear stick of a car. The vibrations from the engine are causing forced oscillations of these objects.



Which of the following are **FREE** oscillations and which are **FORCED** ?

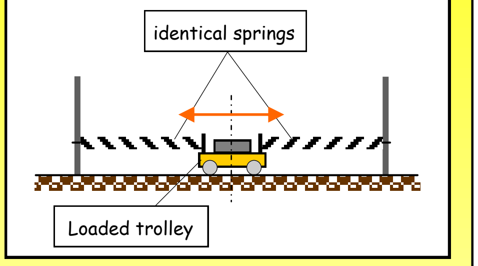
- The wing beat of a mosquito.
- The pendulum movement in a Grandfather clock.
- The vibrations of a cymbal after it has been struck.
- The shaking of a building during an earthquake.
- The vibration of a bat after a cricket ball is struck.
- The vibrations of a washing machine during its spin cycle.

OBSERVING OSCILLATIONS

1. Mass-Spring System

A heavily-loaded trolley is attached by identical springs to two fixed retort stands.

When the trolley is pulled horizontally to one side and released, it is seen to oscillate freely back and forth along the bench.



As the trolley oscillates, the springs are alternately stretched and compressed and we see that the trolley's speed is :

- **GREATEST** at the **CENTRE** of the oscillation.
- **ZERO** at the **EXTREMITIES** of the oscillation.

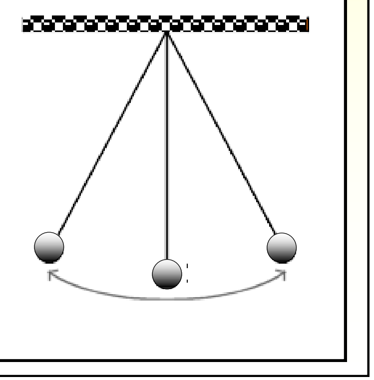
This means that at the **CENTRE** of the oscillation the **KINETIC ENERGY** of The system is a **MAXIMUM** and the **POTENTIAL ENERGY** is a **MINIMUM**.

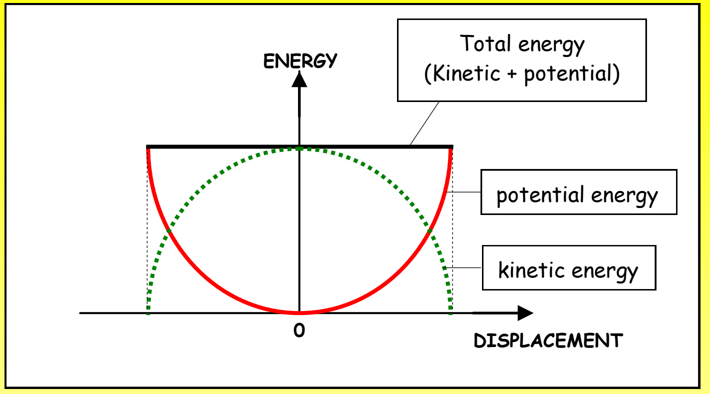
At the **EXTREMITIES** of the oscillation, the **POTENTIAL ENERGY** is a **MAXIMUM** and the **KINETIC ENERGY** is **ZERO**.

2. Simple Pendulum

When the pendulum bob is pulled slightly to one side and released, the pendulum oscillates freely at its **NATURAL FREQUENCY**.

The speed of the bob is a **MAXIMUM** at the **CENTRE** and **ZERO** at the **EXTREMITIES** of the oscillation.





So, as the pendulum oscillates about its equilibrium position, the **KINETIC ENERGY** of the system is a **MAXIMUM** at the **CENTRE** of the oscillation and **ZERO** at the **EXTREMITIES**, whilst the **POTENTIAL ENERGY** is a **MINIMUM** at the **CENTRE** and a **MAXIMUM** at the **EXTREMITIES**.

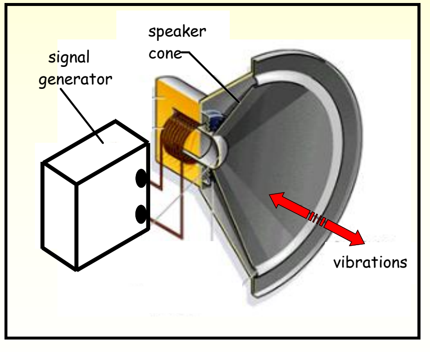
At any point in the oscillation, the **TOTAL ENERGY** of the system is the sum of the **KINETIC** and **POTENTIAL ENERGY** at that point.

3. Vibrating Loudspeaker Cone

A signal generator set at low frequency is used to drive the loudspeaker.

By setting the frequency very low, the motion of the cone, oscillating about its fixed equilibrium position, is clearly seen.

This is a good example of **FORCED OSCILLATION**.

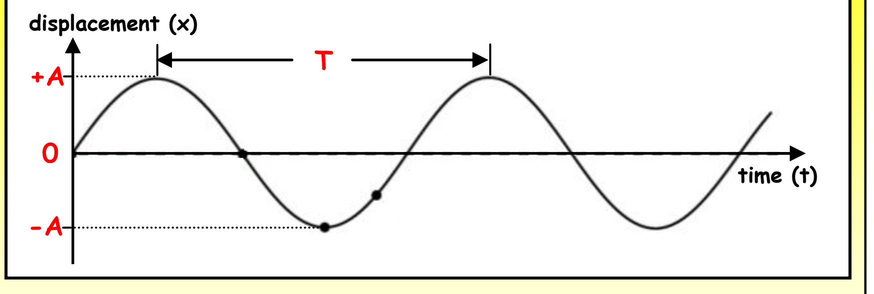


NOTE

In each of the oscillation examples considered, the pattern of motion is :

- **POSITIVE ACCELERATION** when moving **TOWARDS** the **EQUILIBRIUM POSITION**.
- **MAXIMUM SPEED (ZERO ACCELERATION)** at the **EQUILIBRIUM POSITION**.
- **NEGATIVE ACCELERATION (DECELERATION)** when moving **AWAY FROM** the **EQUILIBRIUM POSITION**.
- **ZERO SPEED (MAXIMUM ACCELERATION)** at the **EXTREMITIES** of the oscillation.

GRAPHICAL ANALYSIS OF OSCILLATIONS



The motion of many oscillating systems may be represented by a **DISPLACEMENT/TIME** *graph as seen in the diagram above, which shows The* **SINUSOIDAL** *graph shape characteristic of* **SIMPLE HARMONIC MOTION (SHM).**

Examples of SHM : Swinging pendulum; Oscillating mass-spring system; Vibrating loudspeaker cone; Vibrations of atoms or molecules in a solid.

The equation for a **SINUSOIDAL** *oscillation is :*

$$x = A \sin \omega t = A \sin(2\pi f t)$$

OR

$$x = A \cos(2\pi f t)$$

- **x** = Displacement from the equilibrium position at any time (t).
- **A** = Maximum displacement from the equilibrium position.
- **ω** = Angular frequency of the oscillation.
- **f** = Frequency of the oscillation.

The **MAXIMUM SPEED (v_{max})** *of a simple harmonic oscillator is :*

$$v_{max} = \pm(2\pi f)A$$

DEFINITION OF TERMS USED IN OSCILLATIONS

DISPLACEMENT (x)/m is the distance moved by an oscillating object in either direction from the equilibrium position at any given time.

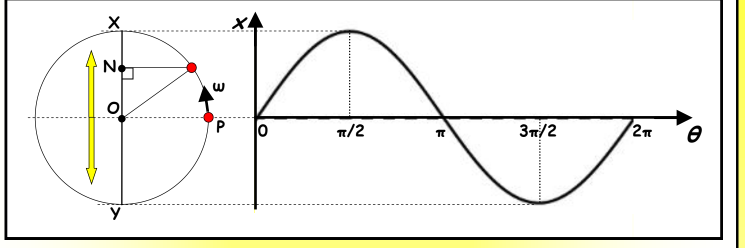
AMPLITUDE (A)/m is the maximum displacement of an oscillating object from the equilibrium position.

PERIOD (T)/s is the time taken for each complete oscillation (i.e. for the oscillating object to go from one side to the other and back again.

FREQUENCY (f)/Hz is the number of complete oscillations per second which the oscillating object undergoes.

ANGULAR FREQUENCY (ω)/rad s-1 is the frequency of the oscillations expressed in radians per second.

This requires some further explanation. Oscillations and circular motion are closely related.



Consider an object **P** moving in a circle with angular frequency **(ω)**. As **P** moves from its starting position and undergoes **1 complete revolution**, the foot of an imaginary perpendicular, **N** from it onto the diameter **XY** performs a simple harmonic motion which takes it from **O → X → O → Y → O**.

When the **linear displacement (x)** of the foot of the perpendicular from P is plotted against its **angular displacement (θ)**, a sine curve is obtained.

$$\text{angular frequency } (\omega) = \frac{\text{angular displacement } (\theta)}{\text{time taken } (t)}$$

So, for **1 complete revolution of P**, which is **1 complete oscillation of the foot of its perpendicular across XY** :

$$\omega = 2\pi/T$$

$$\omega = 2\pi f \quad \text{(since } T = 1/f)$$

(rad s-1) (Hz)

PHASE is the term used to describe the point that an oscillating object Has reached within the complete cycle of an oscillation.

PHASE DIFFERENCE between two oscillations tells us the amount by Which they are 'out of step' (out of phase) with each other.

- Two points that have exactly the same pattern of oscillation are said to be **IN PHASE (i.e. phase difference between them is zero).**
- If the patterns of movement at the two points are exactly opposite to each other, the oscillations are said to be in **ANTIPHASE (i.e. phase difference between them = π radians).**
- If the patterns of movement at the two points has a **phase difference = 2π radians,** the oscillations are **IN PHASE** again because 2π radians is equivalent to one complete revolution or oscillation.

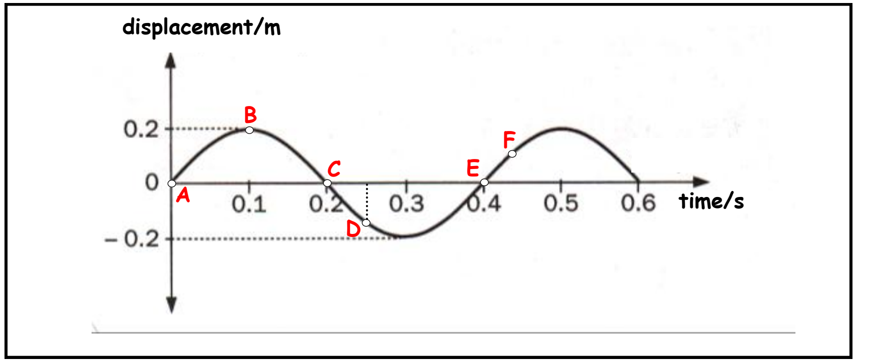
- **PRACTICEQUESTIONS (1)**

1 A mass is suspended from the lower end of a vertical spring whose other end is fixed. The mass is then set into vertical oscillations by displacing it downwards by a distance of **40 mm** and releasing. If it takes **8.4 s** to undergo **18** complete oscillations, calculate :

(a) Its **time period.**

(b) Its **frequency** of oscillation.

2 The displacement/time graph shown below is that for an object performing **simple harmonic motion.**



Several points on the graph have been labelled **(A, B, C, D, E, F).**

(a) Which point(s) is/are : (i) At the **amplitude** of the oscillation ?

(ii) **One period** apart ?

(iii) In **antiphase ?**

(iv) In **phase ?**

(b) Use the graph to find : (i) The **period.**

(ii) The **frequency.**

(iii) The **angular frequency,** of the oscillation.

(c) What is the **phase difference in radians** between points :

(i) **A and B,** (ii) **A and C,** (iii) **C and D,** (iv) **B and E ?**

DEFINITION OF SIMPLE HARMONIC MOTION

SIMPLE HARMONIC MOTION (SHM)

Is the oscillatory motion of an object in which the Acceleration (a) is :

- **Directly proportional to its displacement (x) from a fixed point.**
- **Always in the opposite direction to the displacement**

The general mathematical equation which defines SHM is :

$$a = -\omega^2 x = -(2\pi f)^2 x$$

(acceleration) (angular frequency) (frequency) (displacement)

maximum displacement, x_{max} = ±A (where A = amplitude)

When $x_{max} = +A$, $a = -(2\pi f)^2 x$

When $x_{max} = -A$, $a = +(2\pi f)^2 x$

When $x = 0$, $a = 0$

NOTE : The TIME PERIOD, T = 2πf is independent of the AMPLITUDE of the oscillations.

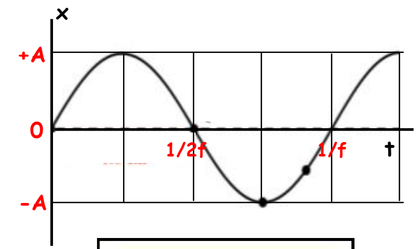
SOLUTIONS TO THE SHM EQUATION

For an object undergoing SHM at **frequency (f)**, the acceleration (a) at displacement (x) is given by : $a = -(2\pi f)^2 x$

The variation of **displacement (x)** with **time (t)** depends on its initial displacement (i.e. the displacement when **t = 0**).

If **x = 0** when **t = 0** (i.e. if the oscillation starts at the **centre (equilibrium position) of the motion** and the object is moving to a maximum displacement = **+A**, then its displacement at time **(t)** is given by :

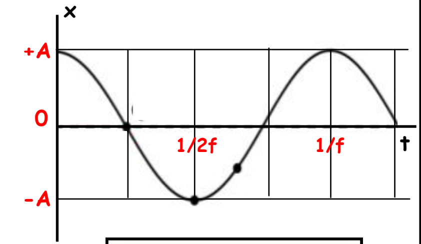
$$x = A \sin(2\pi ft)$$



$x = A \sin(2\pi ft)$

If x = +A when **t = 0** (i.e. If the oscillation starts at the **end or extremity of the motion**, then its displacement at time **(t)** is given by :

$$x = A \cos(2\pi ft)$$



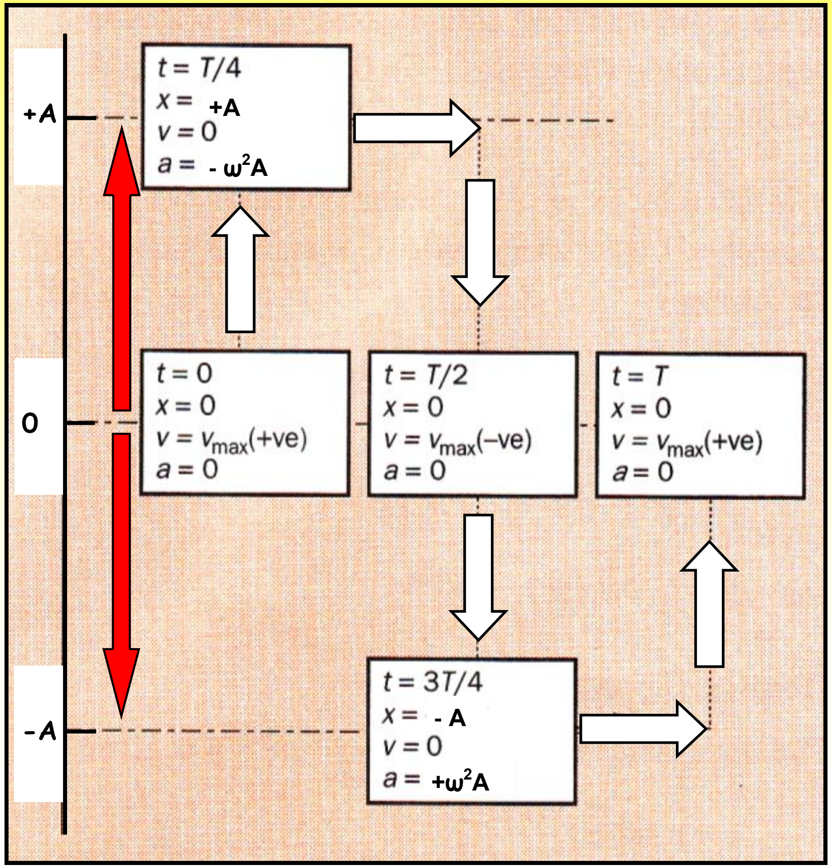
$x = A \cos(2\pi ft)$

NOTE : The quantity **(2πft)** is in **RADIANS**, so make sure your calculator is set to **RAD** for any calculations.

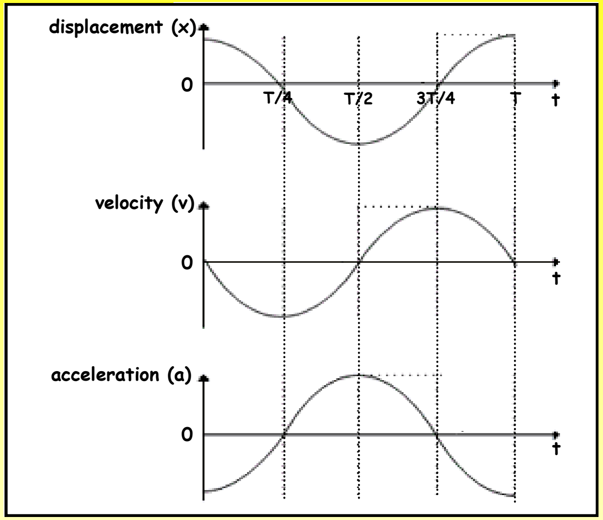
SIMPLE HARMONIC MOTION SUMMARY

Consider an object undergoing SHM about an equilibrium position **(O)**. The period of the oscillation is **(T)** and its amplitude is **(A)**.

The diagram below summarises the values of :
displacement (x) **velocity (v)** **and acceleration (a)** at time **(t)** between **0** and **T**.



DISPLACEMENT, VELOCITY AND ACCELERATION GRAPHS FOR AN OBJECT UNDERGOING SIMPLE HARMONIC MOTION



- These three graphs represent the motion of an object undergoing **Simple harmonic motion**.
- The **v/t** graph can be deduced from the **x/t** graph. This is because : **v = dx/dt (i.e. velocity = gradient of the displacement/time graph).**

position	x	dx/dt	v
equilibrium (centre of oscillation)	0	max^m	max^m
maximum displacement (extremities of oscillation)	±A	zero	zero

v is +ve when dx/dt is +ve (this corresponds to the part of the Oscillation when the object is moving to the right).

v is -ve when dx/dt is -ve (this corresponds to the part of the Oscillation when the object is moving to the left).

- The **a/t** graph can be deduced from the **v/t** graph. This is because : **a = dv/dt (i.e. acceleration = gradient of velocity/time graph).**

position	x	v	dv/dt	a
equilibrium (centre of oscillation)	0	max^m	zero	zero
maximum displacement (extremities of oscillation)	±A	zero	max^m	max^m

- Comparing the **displacement/time (x/t)** and **acceleration/time (a/t)** shows that :
  - They are both **SINE** curves, but the a/t graph is **inverted** relative to the x/t graph.
  - The **acceleration** is always in the **OPPOSITE DIRECTION** to the **displacement**.

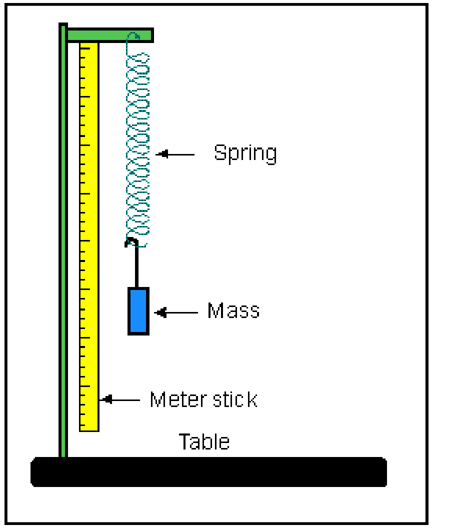
- **PRACTICE QUESTIONS (2)**

1 (a) **State** the general equation for an object undergoing **SHM** and **define** any symbols used.

(b) If the object undergoing SHM goes through **4** complete oscillations in **1 s**, calculate :

(i) The **period (T)**.
(ii) The **frequency (f)**.
(iii) The **angular frequency (ω)**, of the motion.

2 A small mass attached to the end of a fixed vertical spring is pulled down **25 mm** from its equilibrium position and released.



It then undergoes **SHM** with a time period of **1.5 s**.

At time, **t = 0** the mass passes through the equilibrium position **moving upwards**.

(a) What is the **displacement** and **direction of motion** of the mass :

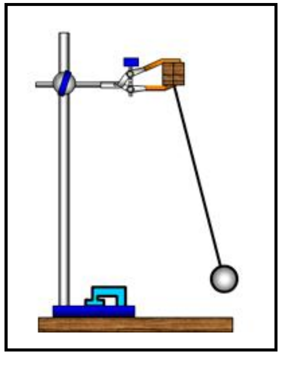
(i) ¼ cycle later (ii) ½ cycle later (iii) ¾ cycle later ?

(b) Calculate the **frequency** and **angular frequency** of the motion.

(c) Calculate the **acceleration** of the mass when its displacement is :

(i) **10 mm** (ii) **25 mm.**

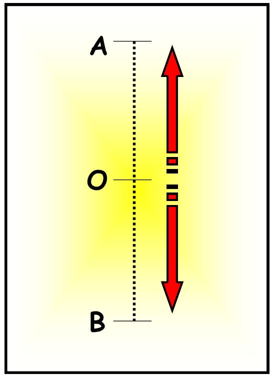
3 A pendulum oscillates with a frequency of **1.5 Hz** and amplitude **12 mm.**



If it is passing through the **midpoint** of its oscillation at time, **t = 0,** write down an equation to represent its displacement in terms of **amplitude, frequency** and **time.**

Use the equation to calculate the **displacement** when t = **0.5 s.**

4 A particle performs **SHM** along a straight line **AOB**, where **O** is the **equilibrium position** and **A** and **B** are the two **extremities** of the motion, equidistant from O.



(a) What can be said about the **direction of the particle's acceleration** as it moves through one cycle ?

(b) When is the **velocity** : (i) **Maximum ?**
(ii) **zero ?**

(c) When is the **acceleration** : (i) **Maximum ?**
(ii) **Zero ?**

(d) When is the **kinetic energy a maximum** and when is the **potential energy a maximum ?**

(e) What can you say about the **total energy** of the system at any point in the oscillation ?

5 The vibration of a component in a machine is represented by the equation :

$$x = 0.3 \text{ mm} \times \sin(2\pi \times 120 \text{ Hz} \times t)$$

What are the values the **amplitude**, **frequency**, **angular frequency** and **period** of this vibration ?

6 A short pendulum oscillates with **SHM** such that its **acceleration (a)** in **m** s^{-2} is related to its **displacement (x)** in **m** by the equation :

$$a = -400\,x$$

What is the **angular frequency** and **frequency** of the pendulum's oscillation ?

7 A mass on a spring oscillates with **SHM** of frequency **1.4 Hz**.

(a) Write an equation of the form $a = -(2\pi f)^2 x$ to show how the mass's acceleration depends on its displacement.

(b) Calculate the acceleration of the mass when it is displaced **5 cm** from the midpoint of its oscillation.

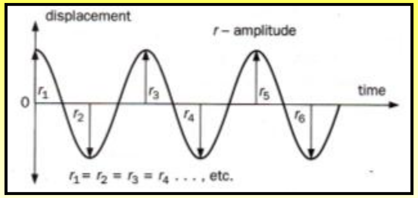
8 A trolley is at rest, tethered between two springs. It is pulled **20 cm** to one side and, when **time, t = 0,** it is released so that it oscillates back and forth. The period of its motion is 2.0 s.

(a) Assuming that its motion is **SHM, write down an equation** to represent the motion.

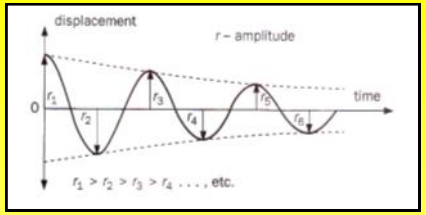
(b) **Sketch a graph to show two cycles** of the motion, giving values on both axes as appropriate.

DAMPED OSCILLATIONS

In theory, **free oscillations** can go on indefinitely (i.e. the amplitude does not decrease with time). In practice however, there are opposing forces (e.g. friction, air resistance) present which dissipate the energy of the system to the surroundings as thermal energy. This causes the amplitude of the oscillations to decrease with time. This effect is called **DAMPING**.

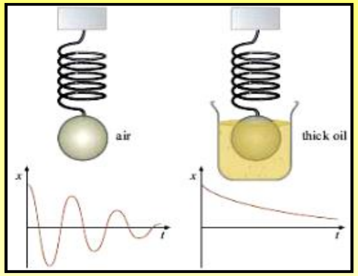


In the presence of dissipative forces, the motion of an oscillating object is **DAMPED**.



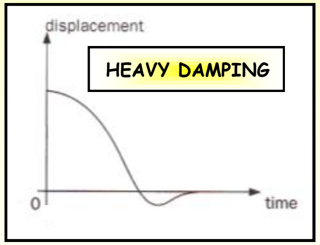
The diagram opposite shows how the amplitude of the oscillations gradually decreases with time in a **LIGHTLY** or **MODERATELY** damped oscillating system.
It should be noted that as the amplitude decreases, the time period remains constant.

The greater the degree of damping, The faster the amplitude decreases.
So, for example, in a mass-spring system oscillating in air the amplitude decreases gradually because the damping is **LIGHT**. The same system oscillating in thick oil is **HEAVILY** damped. This means that the dissipative forces are much greater and so the amplitude decreases much rapidly than in air.

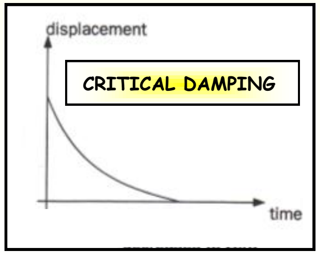


DEGREE OF DAMPING

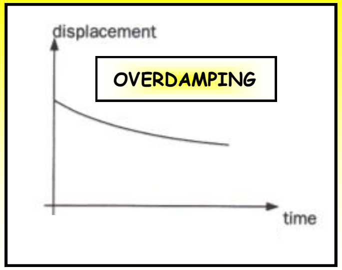
In <u>**HEAVY DAMPING**</u>, the amplitude of oscillation decreases to zero very rapidly. When released, the oscillating system barely overshoots the equilibrium position before coming to rest.



In <u>**CRITICAL DAMPING**</u>, the amplitude of the oscillation decreases to zero in the shortest possible time and does not overshoot the equilibrium position.



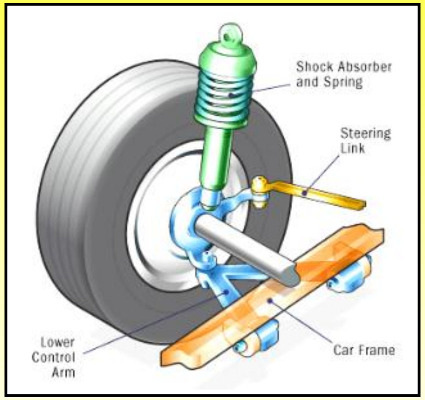
In an <u>**OVERDAMPED**</u> system, there is a very slow return to the equilibrium position.



EXAMPLES OF DAMPING

CAR SUSPENSION SYSTEM

Springs fitted between the wheel axle and the chassis are used to absorb jolts caused by bumps in the road. The springs are damped by shock absorbers (oil dampers) which dissipate the energy of the oscillations.



The dampers provide **CRITICAL DAMPING**, so that after a jolt, the car returns to its equilibrium position in the shortest possible time, with little or no oscillation. In this way the wheels follow an uneven surface, while the car itself follows a virtually horizontal path.

<u>NOTE</u> : If the damping were **HEAVY**, the shock of each bump would be transmitted to the passengers and, if it were **LIGHT**, the car and passengers would bounce around for some time after each bump.