- ***Candidates should be able to*** :

  - *Define the **radian**.*

  - *Convert angles from **degrees into radians** and vice versa.*

  - ***Explain** that a force perpendicular to the velocity of an object will make the object describe a circular path.*

  - ***Explain** what is meant by **centripetal acceleration** and **centripetal force**.*

  - ***Select and apply** the equations for :*

    Speed : $$v = \frac{2\pi r}{T}$$

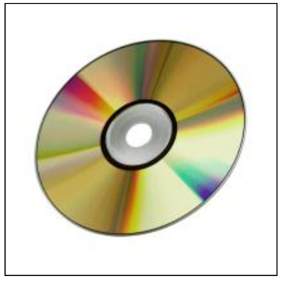
    Centripetal acceleration : $$a = \frac{v^2}{r}$$

  - ***Select and apply** the equation for :*

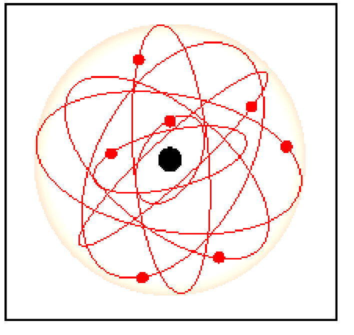
    Centripetal force : $$F = ma = \frac{mv^2}{r}$$

***Circular motion** is an integral part of our everyday experience. Most of our transport makes use of the wheel to convert rotational into linear motion.*

Music and film is readily available to us courtesy of spinning CD's and DVD's, and many fairground rides, such as the Big Wheel, thrill us by giving us a real feel of the centripetal force.



Circular motion is as common to our description of the atom as it is to that of the motion of planets and galaxies.

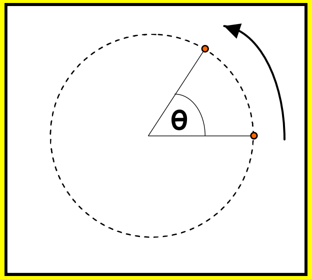


MEASURING ROTATION

ANGULAR DISPLACEMENT

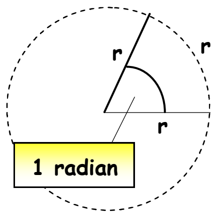
*This is the angle **(θ)** through which an object turns when it is moving in a circle.*

(The equivalent quantity in LINEAR motion is the **LINEAR DISPLACEMENT, s**).



***θ** may be expressed in **DEGREES**, but it is more usually expressed in **RADIANS**.*

One **RADIAN** *is the angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.*



angle in radians = length of arc / Radius

$$\theta = \frac{s}{r}$$

Therefore, for a complete circle : $\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi$

$$360° = 2\pi \text{ radians}$$

Therefore,

$$1 \text{ radian} = \frac{360°}{2\pi} \approx 57.3°$$

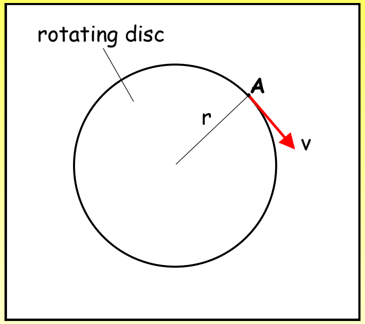
To convert ° to radians - x by π/180°

To convert radians to ° - x by 180°/π

UNIFORM CIRCULAR MOTION

*Is said to occur when an object rotates at a **CONSTANT** or **STEADY** rate.*

Consider a point **A** on the perimeter of a disc of radius **(r)** which is rotating at a steady or constant speed.



Linear speed **(v)** of point A is given by :

$$v = \frac{\text{distance moved in one revolution}}{\text{time taken for one revolution}} = \frac{2\pi r}{T}$$

And since **T = 1/f,** $v = \frac{2\pi r}{1/f} = 2\pi r f$

$$v = \frac{2\pi r}{T} = 2\pi r f$$

v (m s^{-1}), T (period in s), r (m), f (frequency in s^{-1} or Hz)

UNIT G484	Module 2	4.2.1	Circular Motion

- **PRACTICE QUESTIONS (1)**

1 Calculate the ***angular displacement*** of the tip of the ***minute*** hand on a watch in (i) ***Degrees*** and (ii) ***Radians,*** in a time of :

(a) ***5 minutes,*** (b) ***15 minutes,*** (c) ***1 hour.***

2 (a) Calculate the number of ***radians*** in : (i) ***60°***, (ii) ***145°.***

(b) Calculate the number of ***degrees*** in : (i) ***0.8 radian.***
(ii) ***π/4 radian.***

(c) Express ***30°, 60°*** and ***90°*** as multiples of π radians.

3 The wheels on a racing car turn at a frequency of ***10 Hz.*** Calculate :

(a) The ***time period (T).***

(b) The ***angular displacement in radians*** in a time of :

(i) ***25 ms,*** (ii) ***100 ms.***

4 At some point in the past, when the Earth was in its initial stages of formation, it took ***18 hours*** to complete one revolution about its axis. Given that the Earth's diameter is ***12800 km,*** calculate :

(a) The ***speed of rotation*** of a point on the equator.

(b) The ***angular displacement*** of this point in a time of ***30 minutes,*** (i) in ***radians*** and (ii) in ***degrees.***

5 Cyclists racing in the Olympic Velodrome often reach speeds of ***18 m s^{-1}*** on bikes having wheels of diameter ***700 mm.***

Calculate :

(a) The ***time taken for one complete revolution*** of the wheels.

(b) (i) The ***rotational frequency*** of the wheels.

(ii) The ***number of complete revolutions*** made by the wheels in ***4 minutes.***

(iii) The ***distance travelled*** by the cyclist in ***4 minutes.***

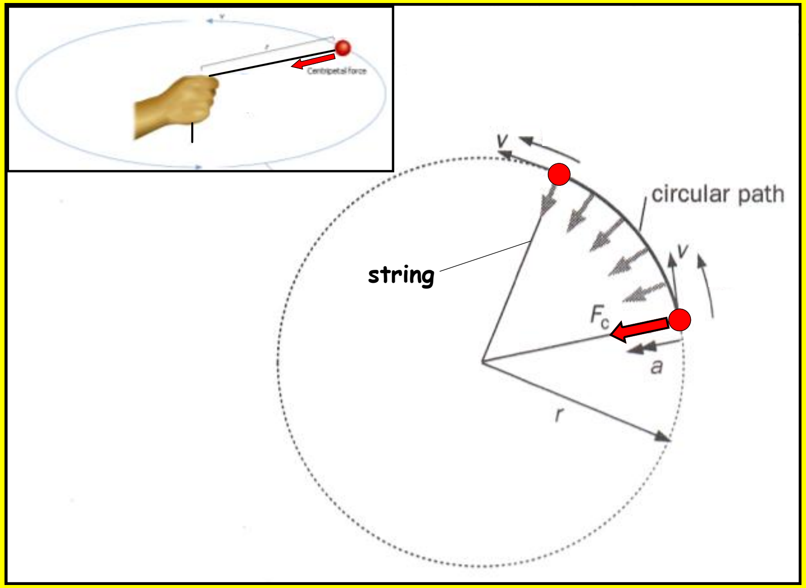
6 The Earth has an orbits the Sun at an average radius of ***1.5 x 10^{11} m.*** Given that it completes its orbit in ***365.3 days***, calculate the Earth's orbital :

(a) ***Frequency.***

(b) ***Linear speed.***

(c) ***Angular speed in radians per second.***

CENTRIPETAL ACCELERATION AND FORCE



Consider a ball attached to the end of a string and whirled in a horizontal circle at **constant speed (v).**

- *The ball's velocity is always directed along the tangent to the circle (i.e. at* **90°** *to the string).*
- *Since the* **speed is constant**, *the* **magnitude of the velocity stays the same,** *but the* **direction of the velocity is continually changing.**
- *Since a change in direction constitutes an acceleration, the ball has an acceleration which is directed towards the centre of the circular path.*

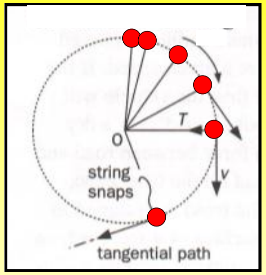
This is the **CENTRIPETAL ACCELERATION (a_c).**

- *According to* **NEWTON'S FIRST LAW**, *an object continues to move in a straight line unless a resultant force acts on it.*

  In this case the resultant force, which is called the **CENTRIPETAL FORCE (F_c),** *acts on the ball towards the centre of the circle giving it a* **CENTRIPETAL ACCELERATION.**

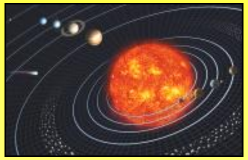
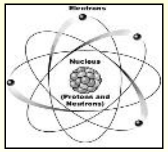
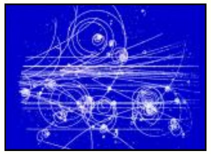
  The centripetal force is provided by the tension in the string.

POINTS TO NOTE

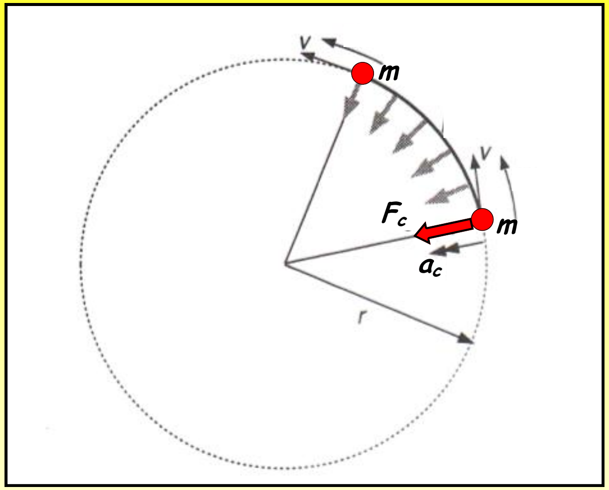


- *If the string breaks, there is no centripetal force and the ball will fly off at a tangent to the circle from the point at which the force stopped acting.*
- *The centripetal acceleration and the centripetal force is directed towards the centre of the circle (i.e. at 90° to the ball's direction of motion).*
- *The centripetal force does* ***NO WORK*** *on the object moving in a circular path. This is because* ***it acts at right angles to the object's direction of motion, so there is no actual motion in the force direction.***

EXAMPLES OF CENTRIPETAL FORCE

Situation in which a centripetal force acts	What provides the centripetal force
A planet in orbit around the Sun. 	The gravitational attraction force acting between the planet and the Sun.
An object on the Earth's surface. 	The force of gravity acting on the object (i.e. its weight).
A car rounding a bend in the road. 	The frictional force acting between the tyres and the road.
An electron in orbit around the nucleus. 	The electrostatic attraction force acting between the negatively charged electron and the positively charged nucleus.
Charged particles moving through a magnetic field acting at right angles to the direction of motion of the particles. 	The magnetic force acting on the charged particles.

CENTRIPETAL FORCE AND ACCELERATION FORMULAE



- For a object of **mass (m)** moving with **constant speed (v)** in a circular path of **radius (r)**, the ***CENTRIPETAL ACCELERATION*** **(a_c)** is given by :

$$a_c = \frac{v^2}{r}$$

(a_c: (m s^{-2})) (v: (m s^{-1})) (r: (m))

UNIT G484	Module 2	4.2.1	Circular Motion

- *According to* ***NEWTON'S 2nd LAW :***

resultant force = mass x acceleration

$$F = m\,a$$

So the ***CENTRIPETAL FORCE (F_c)*** *acting on the object is given by :*

$$F_c = \frac{m\,v^2}{r}$$

(F_c in N, m in kg, v in m s^{-1}, r in m)

- **PRACTICE QUESTIONS (2)**

1 Referring to both ***magnitude*** and ***direction,*** describe how each of the following quantities will change as a body moves in a circular path at constant speed :

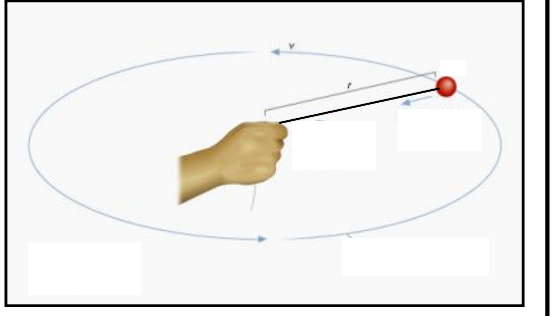
SPEED, VELOCITY, MOMENTUM, KINETIC ENERGY, CENTRIPETAL ACCELERATION, CENTRIPETAL FORCE.

2 Name the force which provides the ***centripetal force*** needed for circular motion in each of the following cases :

(a) A satellite orbiting around the Earth.
(b) A racing car rounding a bend on a flat, rough track.
(c) The weight at the end of a swinging pendulum.
(d) A cart doing the loop the loop in a fairground roller coaster.

3 ***Explain*** what happens when the driver in a car moving along a flat, frozen lake tries to move in a circle by turning the steering wheel. 6

4 A ball of mass ***180 g*** is attached to a string and whirled round in a horizontal circle of radius ***400 mm*** at constant speed.



If the ball completes ***three*** revolutions each second, calculate :

(a) The ***linear speed*** of the ball.

(b) The ***centripetal acceleration*** of the ball.

(c) The ***tension*** in the string.

5 The Earth orbits the Sun with a period of ***365.3 days*** at a mean radius of ***1.5×10^8 km.*** Assuming the orbit to be circular and given that the mass of the Earth is ***6.0×10^{24} kg,*** calculate :

(a) The ***distance travelled*** by the Earth in one complete orbit.

(b) The ***orbital speed*** of the Earth.

(c) The ***centripetal acceleration*** of the Earth.

(d) The ***gravitational force*** acting between the Earth and the Sun.

- **HOMEWORK QUESTIONS**

1 A proton of mass 1.67×10^{-27} *kg* moving at a constant speed of 2.5×10^{7} $m\ s^{-1}$ enters a uniform magnetic field at right angles to its path and as a result it is caused to move in a circular path of radius ***275 mm.*** Calculate for the proton :

(a) The ***time taken*** for 1 complete orbit.

(b) The ***centripetal acceleration.***

(c) The ***force*** exerted on it by the magnetic field.

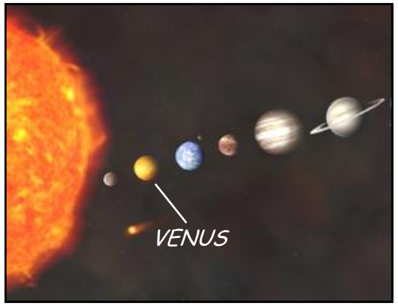
2 An object moves in a circular path at ***constant speed.*** Explain why :

(a) The ***velocity*** of the object is continually changing even though its speed remains constant.

(b) The object ***accelerates*** even though its speed remains constant.

3 The turning circle of an aircraft, when flying horizontally at a constant speed of $600\ m\ s^{-1}$, has a radius of ***65 km.*** Given that the total mass of the aircraft is 1.2×10^{4} *kg,* Calculate the ratio of ***centripetal force to weight*** for the aircraft.
Take $g = 9.81\ N\ kg^{-1}$.

4 Venus orbits the Sun once every ***225 days*** at an average distance of 1.05×10^{11} *m.* Given that the mass of Venus is 4.92×10^{24} *kg,* Calculate :



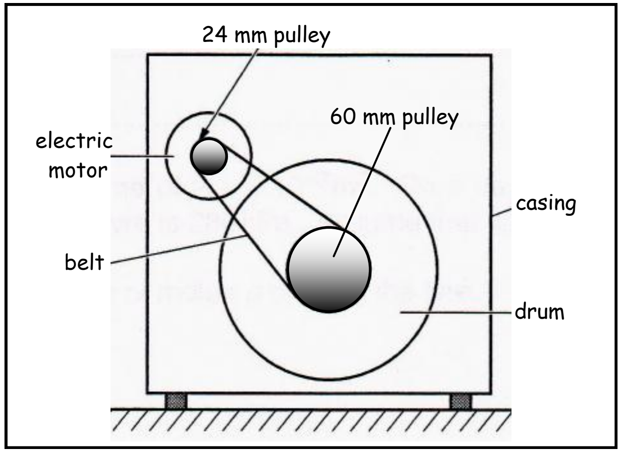
(a) Its ***orbital speed.***

(b) Its ***centripetal acceleration.***

(c) The ***gravitational force*** exerted on Venus by the Sun.

5 A pulley wheel of diameter ***24 mm*** fitted to an electric motor in a machine rotates at a frequency of ***30 Hz.***

A belt fitted to the wheel is used to drive a drum in the machine as shown opposite.



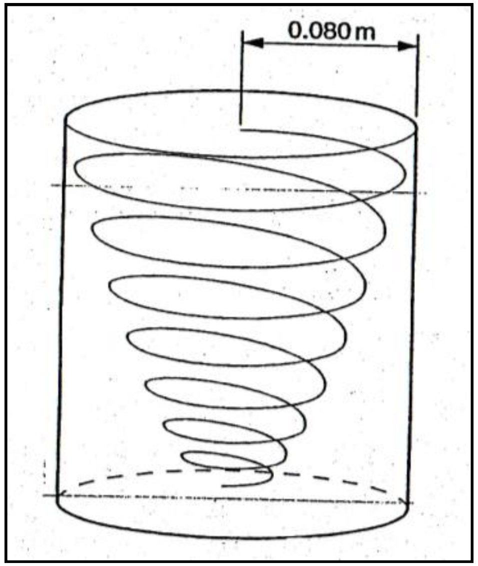
(a) Calculate :

(i) The ***speed*** of the belt on the wheel.
(ii) The ***centripetal acceleration*** of the belt attached to the wheel as it moves round the ***24 mm*** pulley wheel.

(b) The belt drives the drum via another pulley wheel of diameter ***60 mm*** attached to the drum axle. Calculate :

(i) The ***frequency*** of rotation of the ***60 mm*** pulley wheel.
(ii) The **centripetal acceleration** of the belt as it passes round the ***60 mm*** pulley wheel.

6 In one type of vacuum cleaner, dusty air enters a cylindrical container of radius ***0.080 m.*** The air swirls around inside the cylinder, travelling at maximum speed of ***380 m s^{-1},*** as shown in the diagram opposite.



Assume that the maximum speed of dust particles in the air flow occurs for air rotating in a horizontal circle of radius ***0.080 m.***

(a) Show that the acceleration of a dust particle travelling at this maximum speed is ***1.8 x 10^{6} m s^{-2}.***

(b) Calculate the force necessary to give a dust particle, of mass ***5.0 x 10^{-7} kg,*** the acceleration in (a).

(c) The weight of the dust particle is ***4.9 x 10^{-6} N.*** Determine how many times the force in (b) is greater than the weight of the particle.

(d) Suggest what happens to a dust particle in this cleaner.