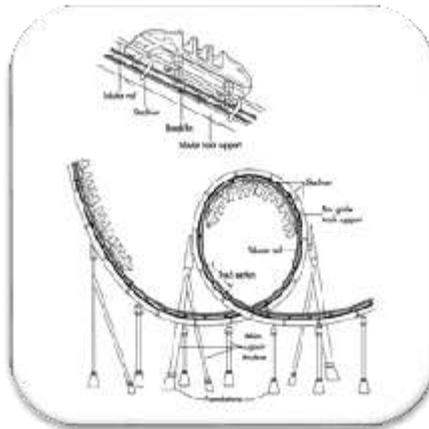


A2 Physics Unit 4

1 Forces & Momentum



KS5 AS PHYSICS 2450

Chapter Map

1.1 Momentum and impulse

Specification link-up 3.4.1: Momentum concepts

How do we calculate momentum?

What is the connection between Newton's first and second laws of motion and momentum?

What is an impulse, and how is it calculated from a force v. time graph?

1.2 Impact forces

Specification link-up 3.4.1: Momentum concepts

What happens to the impact force (and why?) if the duration of impact is reduced?

How do we calculate $\Delta(mv)$ for a moving object which stops or reverses?

What happens to the momentum of a ball when it bounces off a wall?

1.3 Conservation of momentum

Specification link-up 3.4.1: Momentum concepts

Is momentum ever lost in a collision?

What do we mean by *conservation of momentum*?

What condition must be satisfied if the momentum of a system is conserved?

1.5 Explosions

Specification link-up 3.4.1: Momentum concepts

What energy changes takes place in an explosion?

What can we always say about the total momentum of a system that has exploded?

What are the consequences when, after the explosion, only two bodies move apart?

1.4 Elastic and inelastic collisions

Specification link-up 3.4.1: Momentum concepts

What is the difference between an elastic collision and an inelastic collision?

What is conserved in a perfectly elastic collision?

Are any real collisions ever perfectly elastic?

1.4 Elastic & Inelastic Collisions

What is the difference between an elastic collision and an inelastic collision?
(AO1a/ A02a)

What is conserved in a perfectly elastic collision? (AO1a)

How can collisions be used to work out precise bullet velocities? (A01a)

Basic Challenge
E-D

Be able to... define in words the idea of elastic and inelastic collisions.

Medium Challenge
D-B

Be able to... define in words and **apply** using formulae the idea of elastic and inelastic collisions

Harder Challenge
B-A*

Be able to... Extend the ideas into **expressions** to **analyse** situations such as a ballistic pendulum.

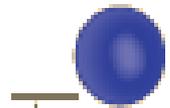
Thinking point dropping a ball....

By conservation of energy:

Energy before = Energy after

$$PE = mgh$$

$$KE = 0$$



h

The beginning energy is all potential energy.

$$mgh$$

$$= \frac{1}{2}mv^2$$

The *m* on both sides tells you that the final velocity doesn't depend upon the mass.

The final energy is all kinetic energy.

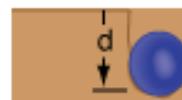
The velocity just before impact is $v = \sqrt{2gh}$

$$KE = \frac{1}{2}mv^2$$

$$PE = 0$$



Greater penetration implies smaller impact force.



Harder ground, less penetration, higher impact force.



If it bounces back, the impact force is even greater because of the greater change in momentum.

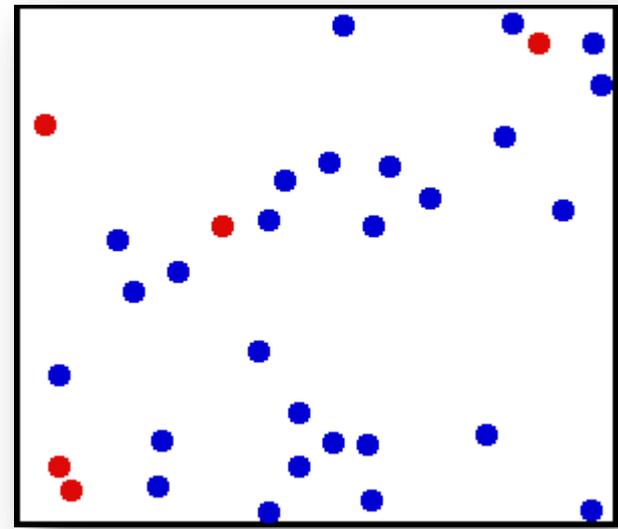
Elastic Collisions

A simple way to look at this concept is to think of the model of atoms which uses round spheres. The collisions of atoms are **elastic** collisions.

An **elastic collision** is a collision in which the total kinetic energy of the colliding bodies after collision is equal to their total kinetic energy before collision.

Elastic collisions occur only if there is **no net conversion** of **kinetic energy** into other forms.

During the collision **kinetic energy** is first converted to **potential energy** associated with a repulsive force between the particles then this potential energy is converted back to kinetic energy when the particles move apart.



This concept is one of the assumptions which underpins the ideal gas law theories.

NB: for objects which are rotating things get more complex i.e. molecules

m

m

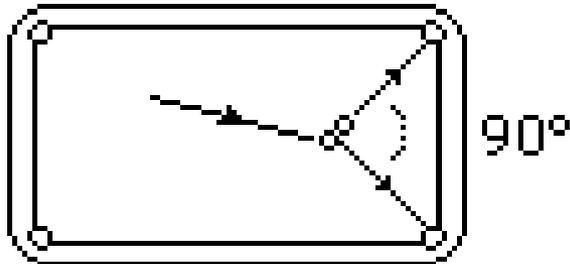
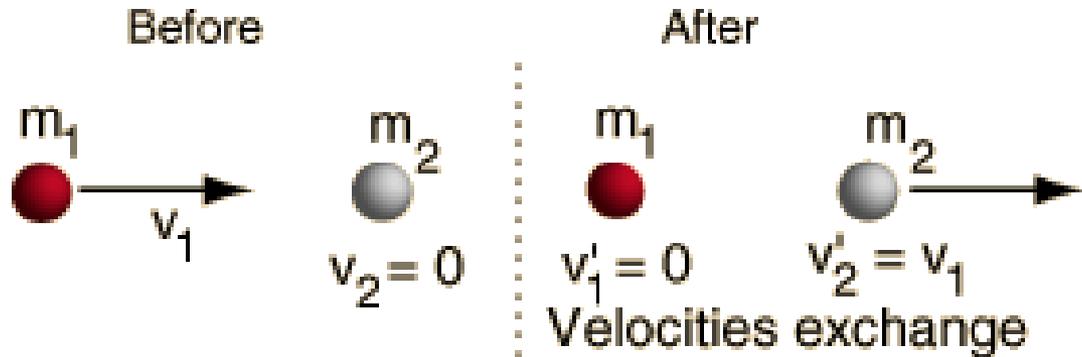
Example Elastic

For a head-on collision with a stationary object of equal mass, the projectile will come to rest and the target will move off with equal velocity, like a head-on shot with the cue ball on a pool table. This may be generalised to say that for a head-on elastic collision of equal masses, the **velocities will always exchange**.

For a non-head-on elastic collision between equal masses, the angle between the velocities after the collision **will always be 90 degrees**. The spot on a pool table is placed so that a collision with a ball on the spot which sends it to a corner pocket will send the cue ball to the other corner pocket.

Case I: $m_1 = m_2$

If not head-on then $\theta = 90^\circ$



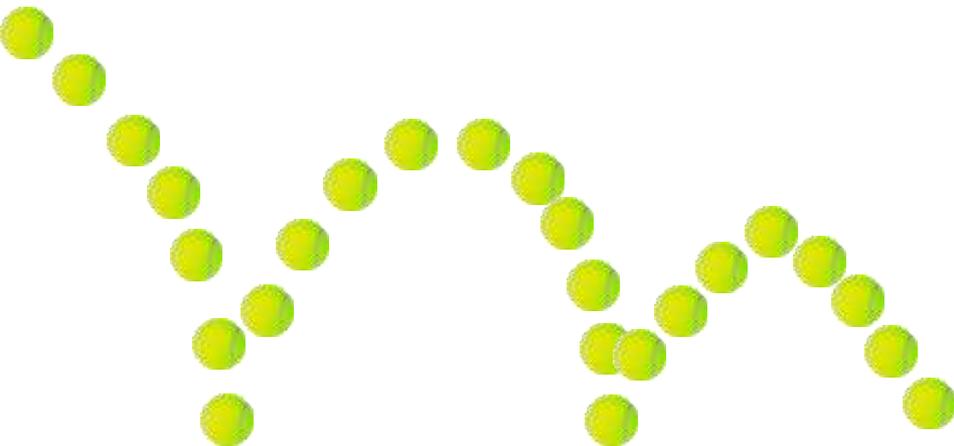
Inelastic Collisions

An **inelastic collision** is a collision in which kinetic energy is not conserved.

In collisions of macroscopic bodies, **some kinetic energy** is turned into **vibrational** energy of the atoms, causing a heating effect, and the bodies are deformed.

The molecules of a gas or liquid **rarely experience perfectly elastic collisions** because kinetic energy is exchanged between the molecules' translational motion and their internal degrees of freedom with each collision. However, averaged across an entire sample, **molecular collisions are elastic**.

Inelastic collisions may not conserve kinetic energy, but they do obey conservation of momentum. They can be partial (items move apart) or perfect (stick together)



This concept is also one of the flaws in ideal gas law theories.



Example Inelastic

Most collisions between objects involve the loss of some kinetic energy and are said to be inelastic. In the general case, the final velocities are not determinable from just the initial velocities. If you know the velocity of one object after the collision, you can determine the other. A completely inelastic collision is one in which objects stick together after the collision, and this case may be analysed in general terms.....

$$m_1 u_1 + m_2 u_2 = m_1 u_1 + 0 \quad \text{P before}$$

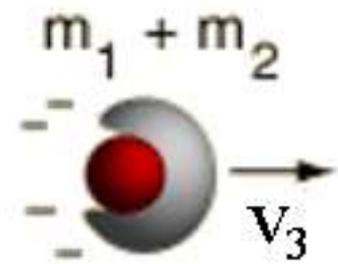
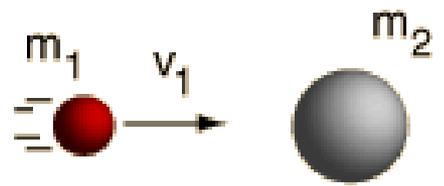
$$(m_1 + m_2) v_3 \quad \text{p after}$$

$$m_1 u_1 = (m_1 + m_2) v_3 \quad \text{p before = p after}$$

NB: the difference between the two KE calculations is the lost KE.

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 u_1^2 + 0 \quad \text{KE before}$$

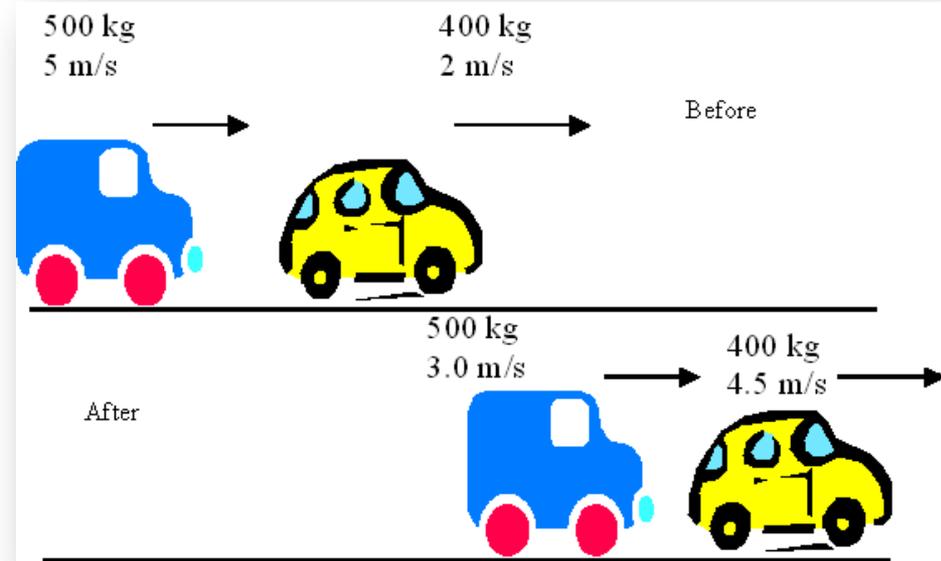
$$\frac{1}{2} (m_1 + m_2) v_3^2 \quad \text{KE after}$$



Example Inelastic

The diagram shows two cars at a fairground, before and after bumping into each other. One car and driver has a total mass of 500 kg, while the other car and driver has a total mass of 400 kg. What is;

1. the total kinetic energy before the collision
2. the total kinetic energy after the collision.
3. the total loss in kinetic energy.



$$\text{Kinetic Energy} = \frac{1}{2} mv^2$$

$$\text{Kinetic Energy of blue car} = \frac{1}{2} \times 500 \text{ kg} \times (5 \text{ m/s})^2 = 6250 \text{ J}$$

$$\text{Kinetic Energy of yellow car} = \frac{1}{2} \times 400 \text{ kg} \times (2 \text{ m/s})^2 = 800 \text{ J}$$

$$\text{Total energy} = 6250 \text{ J} + 800 \text{ J} = 7050 \text{ J}$$

$$\text{Kinetic Energy} = \frac{1}{2} mv^2$$

$$\text{Kinetic Energy of blue car} = \frac{1}{2} \times 500 \text{ kg} \times (3 \text{ m/s})^2 = 2250 \text{ J}$$

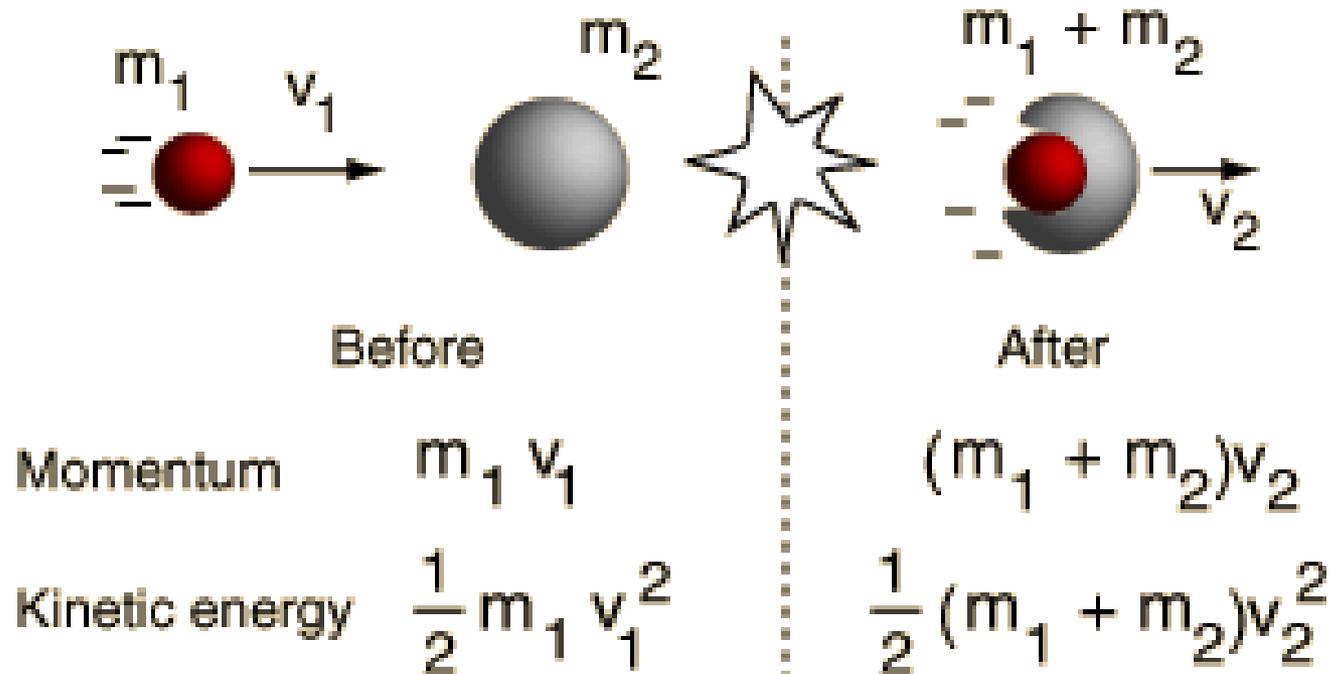
$$\text{Kinetic Energy of yellow car} = \frac{1}{2} \times 400 \text{ kg} \times (4.5 \text{ m/s})^2 = 4050 \text{ J}$$

$$\text{Total energy} = 2250 \text{ J} + 4050 \text{ J} = 6300 \text{ J}$$

$$\text{Total loss} = 7050 \text{ J} - 6300 \text{ J} = 750 \text{ J}$$

Inelastic...

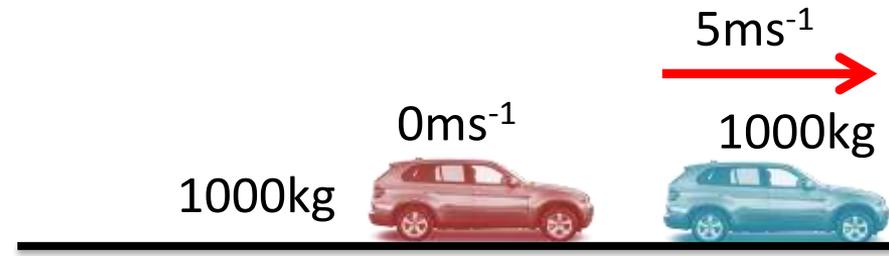
The extreme or perfectly inelastic collision is one in which the colliding objects stick together after the collision, and this case may be analysed in general terms:



From conservation of momentum:

$$m_1 v_1 = (m_1 + m_2)v_2 \Rightarrow v_2 = \frac{m_1}{m_1 + m_2} v_1$$

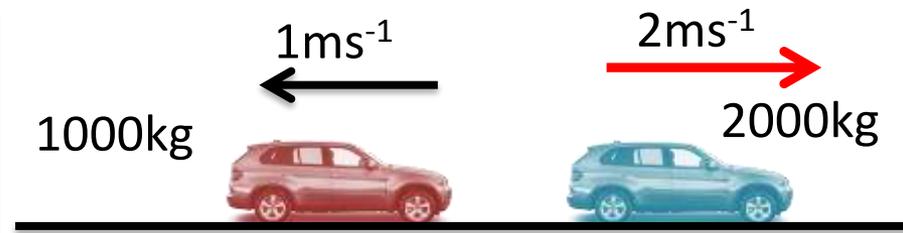
Perfect elastic - collide & transfer all velocity – momentum & KE cons



Perfect inelastic - collide & move together – momentum cons / KE not cons



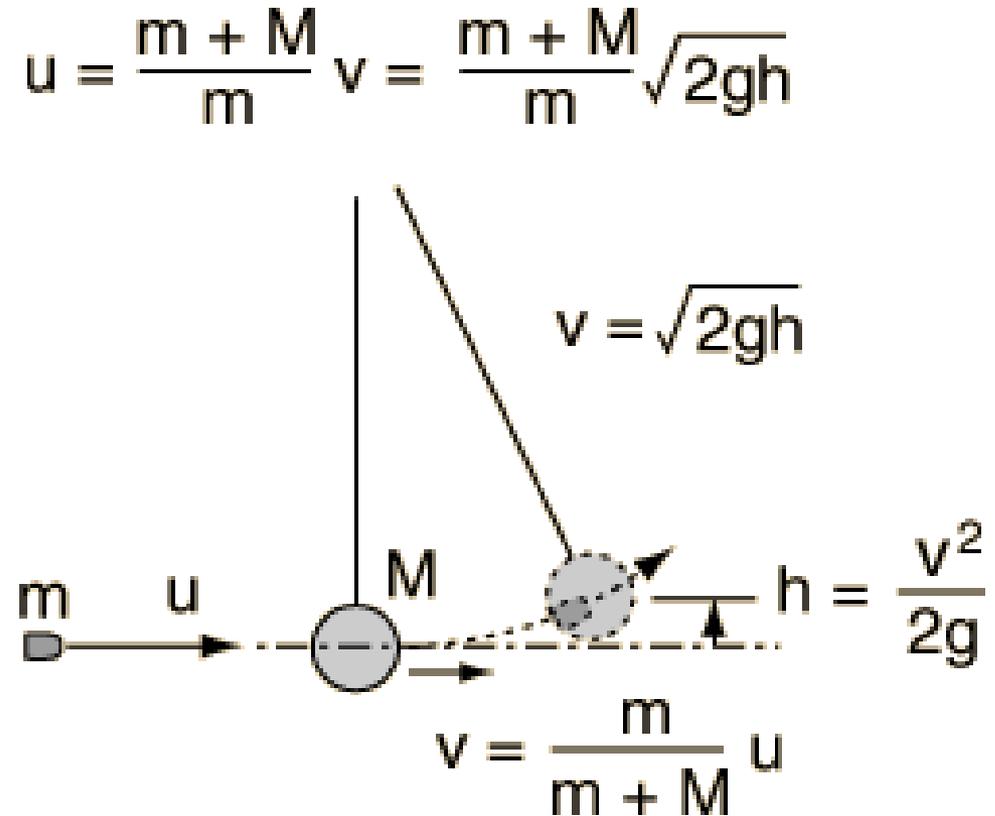
Partial inelastic - collide & repel – momentum cons / KE not cons



Ballistic Pendulum Example... (Perfectly inelastic)

In the back courtyard of the munitions factory hung an old, scarred block of wood. As quality control for the cartridges coming off the assembly line, someone would regularly take a gun to the courtyard and fire a bullet into the block.

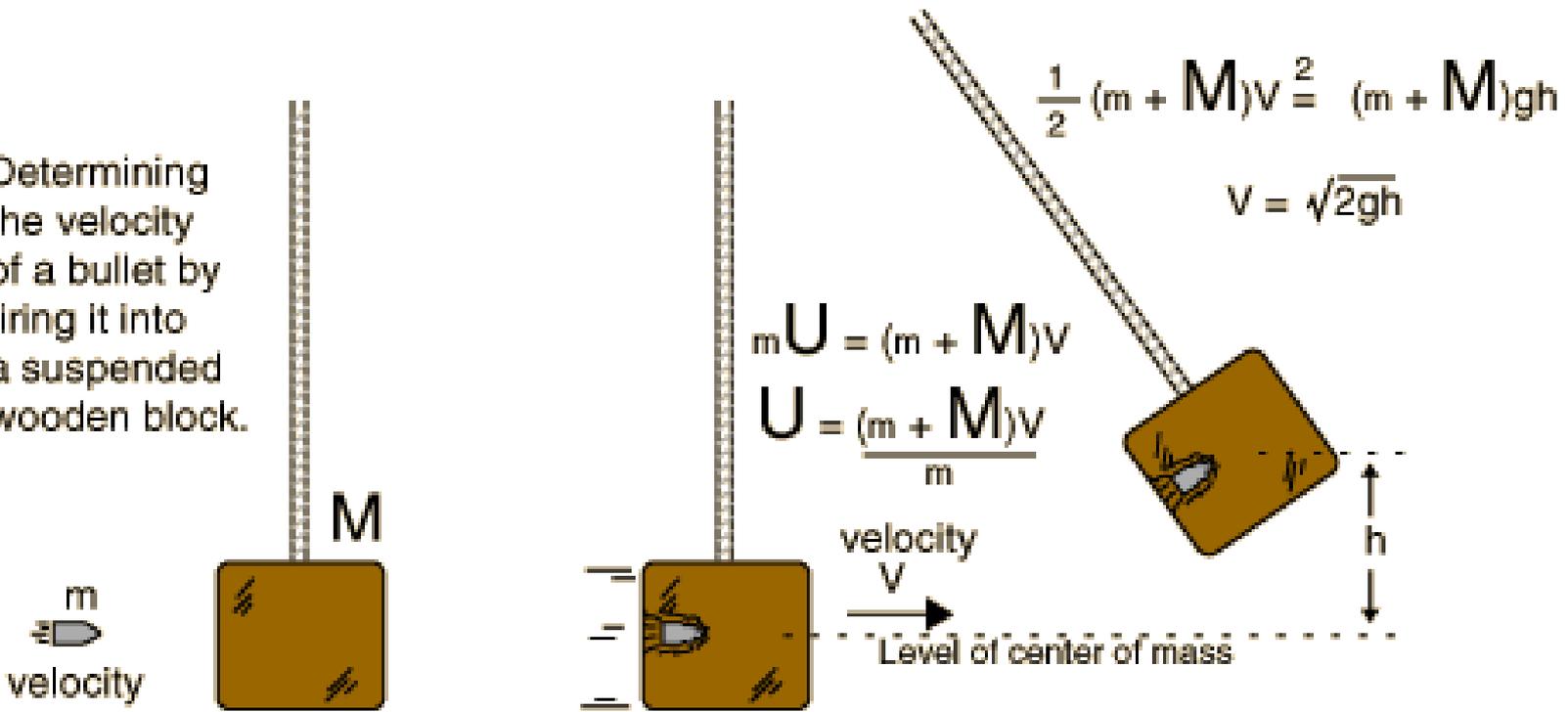
Measuring the height of the swing revealed the speed of the bullet, but since the block was increasing in mass with the added bullets, the mass of the block had to be checked as well as the mass of the bullet being fired.



Look at this example and see if you can get the general idea of what this means, then look at the example which follows....

Ballistic Pendulum Alternative.....

Determining the velocity of a bullet by firing it into a suspended wooden block.



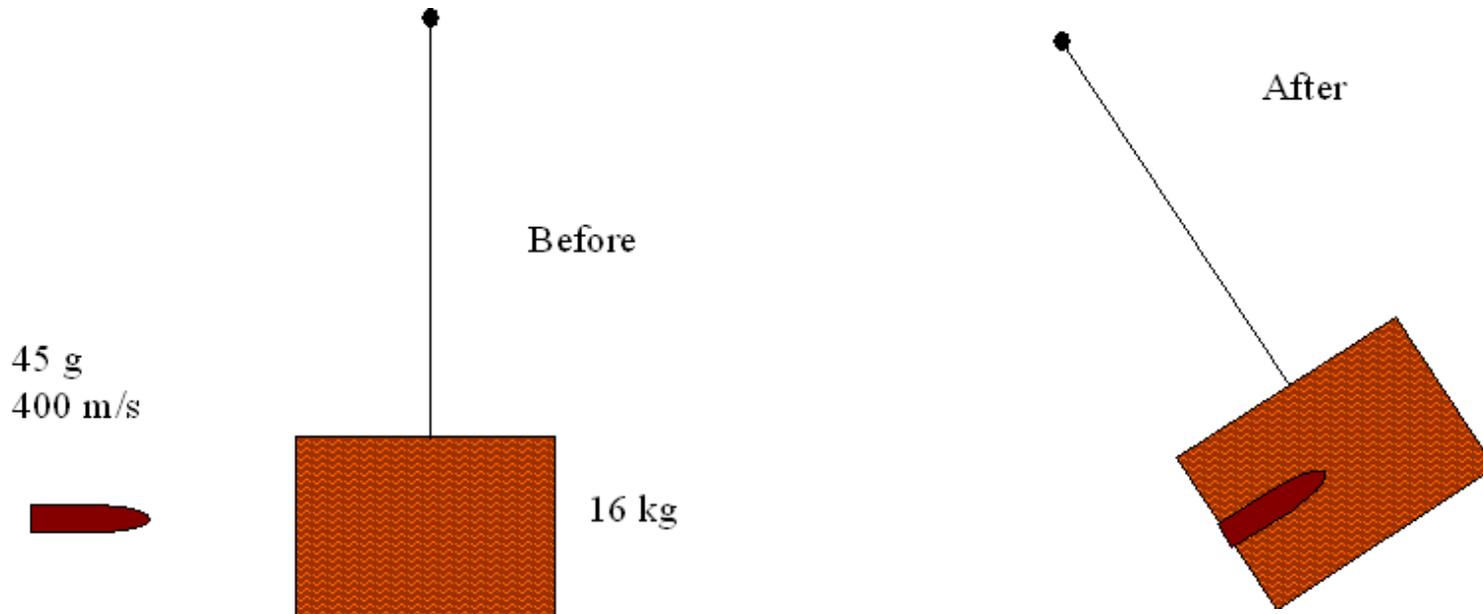
Kinetic energy $\frac{1}{2} mv^2$ is not conserved in the collision. Momentum mv is conserved in the collision

Energy (potential plus kinetic energy) is conserved as the combined masses swing up after the collision.

Perfectly Inelastic Question....

bullet of mass 45 g is travelling horizontally at 400 m/s when it strikes a wooden block of mass 16 kg suspended on a string so that it can swing freely. The bullet is embedded in the block. **Calculate:**

1. The velocity at which the block begins to swing.
2. The height to which the block rises above its initial position.
3. How much of the bullet's kinetic energy is converted to internal energy.



Perfectly Inelastic

bullet of mass 45 g is travelling horizontally at 400 m/s when it strikes a wooden block of mass 16 kg suspended on a string so that it can swing freely. The bullet is embedded in the block. Calculate:

1. The velocity at which the block begins to swing.

Momentum before = momentum after

Momentum before = $m_1 u_1 + 0$

Momentum before = $(0.045 \text{ kg} \times 400 \text{ m/s}) + (0)$

Momentum before = 18 kg m/s

Momentum after = total mass of bullet and wood x speed

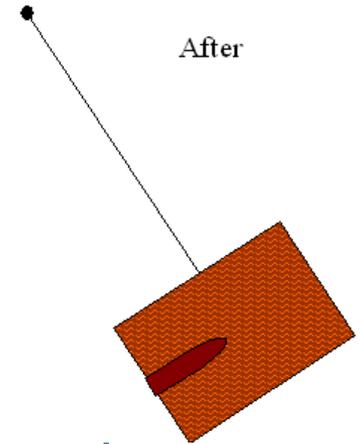
Momentum after = $(16.000 \text{ kg} + 0.045 \text{ kg}) \times v \text{ m/s}$

Momentum after = $16.045 v \text{ kg m/s}$

Therefore:

$16.045 v \text{ kg m/s} = 18 \text{ kg m/s}$

$v = 18 \text{ kg m/s} \div 16.045 = \mathbf{1.12 \text{ m/s}}$



Perfectly Inelastic

bullet of mass 45 g is travelling horizontally at 400 m/s when it strikes a wooden block of mass 16 kg suspended on a string so that it can swing freely. The bullet is embedded in the block. Calculate:

2) height to which the block rises above its initial position.

We use the conservation of energy principle:

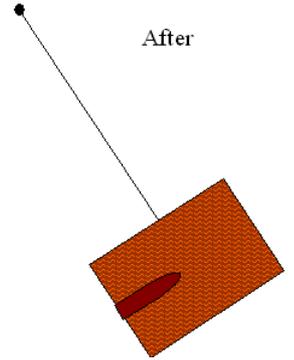
Kinetic energy = potential energy

$$\frac{1}{2} mv^2 = mg\Delta h$$

m's cancel out.

Rearranging:

$$\Delta h = \frac{v^2}{2g} = \frac{1.12^2}{2 \times 9.81 \text{ ms}^{-2}} = \mathbf{0.064 \text{ m}}$$



Perfectly Inelastic

bullet of mass 45 g is travelling horizontally at 400 m/s when it strikes a wooden block of mass 16 kg suspended on a string so that it can swing freely. The bullet is embedded in the block. Calculate:

3) How much of the bullet's kinetic energy is converted to internal energy.

We need to know the kinetic energy of the bullet:

$$E_k = \frac{1}{2} mv^2 = \frac{1}{2} \times 0.045 \text{ kg} \times (400 \text{ m/s})^2 = 3600 \text{ J}$$

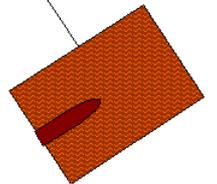
Now we need to know the kinetic energy of the block and bullet:

$$E_k = \frac{1}{2} \times 16.045 \text{ kg} \times (1.12 \text{ m/s})^2 = 10.1 \text{ J}$$

$$\text{Kinetic energy lost} = 3600 \text{ J} - 10.1 \text{ J} = \mathbf{3590 \text{ J}}$$

This energy is not destroyed, but converted into internal energy.

After



Newton's Cradle...



What is happening?

Momentum?

Kinetic Energy?

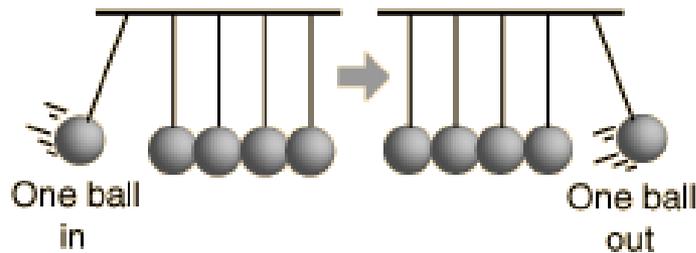
Potential Energy?

Shockwave propagation?

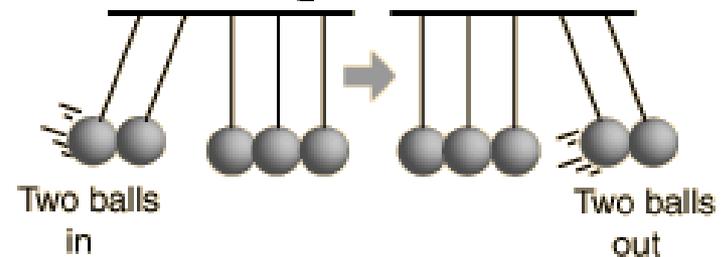
Swinging Balls...

A popular demonstration of conservation of momentum and conservation of energy features several polished steel balls hung in a straight line in contact with each other. If one is pulled back and allowed to strike the line, one ball flies out the other end. If two balls are sent in, two come out, and so forth.

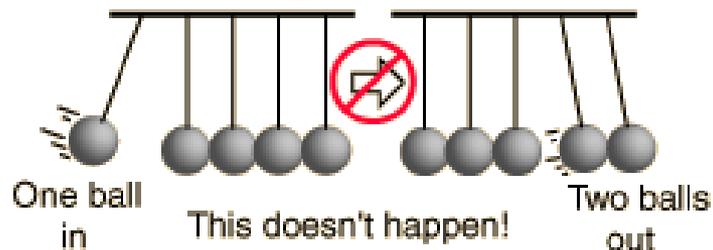
Momentum in: $mv =$ momentum out
 Kinetic energy in: $\frac{1}{2}mv^2 =$ kinetic energy out



Momentum in: $2mv =$ momentum out
 Kinetic energy in: $\frac{1}{2}2mv^2 =$ kinetic energy out



Momentum in: $mv =$ momentum out
 Kinetic energy in: $\frac{1}{2}mv^2 \neq$ kinetic energy out!



Conserving momentum in this case requires that the two balls come out with half the speed.

$$\text{Momentum out} = 2m \frac{v}{2}$$

But this gives

$$\text{Kinetic energy out} = \frac{1}{2} 2m \frac{v^2}{4}$$

Which amounts to a loss of half of the kinetic energy!



1.4 Elastic & Inelastic Collisions

- 1)  a) i) Return to same height indicates elastic collision $KE = mg\Delta h$
ii) Loss of height indicates loss of energy or inelastic

b) $mg\Delta h \neq KE$ $\frac{0.9}{1.2} = 0.75$ eff
Hence 25% lost

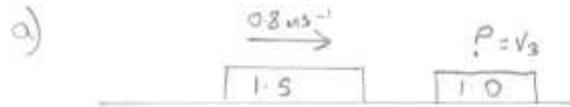
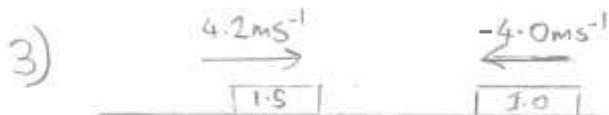


a) $\frac{15 \times 800 + 5 \times 1200}{2000} = 9 \text{ m/s}^{-1}$
(same direction)

b) KE at start = $\frac{1}{2} \times 800 \times 15^2 + \frac{1}{2} \times 1200 \times 5^2$
 $= 90,000 \text{ J} + 15,000 \text{ J}$
 $= 105,000$
 $= \underline{\underline{105 \text{ kJ}}}$

KE at end = $\frac{1}{2} \times 2000 \times 9^2$
 $= 81,000$
 $= 81 \text{ kJ}$

Diff is 24 kJ (wasted)



$$M_1V_1 + M_2V_2 = M_1V_3 + M_2V_4$$

$$4.2 \times 1.5 + (-4 \times 1) = 1.5 \times 0.8 + V_3 \times 1$$

$$4.2 \times 1.5 - 4 - 1.5 \times 0.8 = V_3$$

$$6.3 - 4 - 1.2 = V_3$$

$$\underline{1.1 \text{ ms}^{-1}} = V_3 \text{ (left to right)}$$

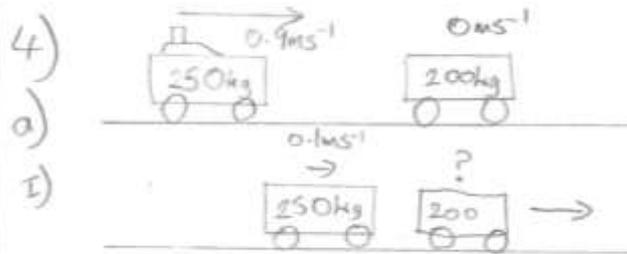
or reverse of original

b)

$$\begin{aligned} \text{Before} &= 0.5 \times 1.5 \times 4.2^2 + 0.5 \times 1 \times 4^2 \\ &= 13.23 \text{ J} + 8 \text{ J} \\ &= 21.23 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{After} &= \frac{1}{2} \times 1.5 \times 0.8^2 + \frac{1}{2} \times 1 \times 1.1^2 \\ &= 0.48 \text{ J} + 0.605 \text{ J} \\ &= 1.085 \text{ J} \end{aligned}$$

$$\text{Diff Wasted} = 20.145 = \underline{\underline{20 \text{ J}}}$$



$$0.9 \times 250 = 0.1 \times 250 + 200V$$

$$\frac{225 - 25}{200} = 1 \text{ ms}^{-1} \text{ (left to right)}$$

i)

$$\begin{aligned} \text{KE before} &= \frac{1}{2} \times 250 \times 0.9^2 \\ &= \underline{\underline{101.25 \text{ J}}} \end{aligned}$$

$$\begin{aligned} \text{KE after} &= \frac{1}{2} \times 0.1^2 \times 250 + \frac{1}{2} \times 200 \times 1^2 \\ &= 1.25 \text{ J} + 100 \text{ J} \\ &= \underline{\underline{101.25 \text{ J}}} \quad \text{Hence KE is conserved} \end{aligned}$$

b) Driver 250 kg car exp force which slows her down. Other driver accelerates in similar way